Applied Mathematics 3B

Assignment #10 **7:30, 25 October 2011**

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1. Let
$$\{\, |0\rangle, \, |1\rangle, \, |2\rangle, \, |3\rangle \,\}$$
denote an orthonormal basis in $\mathbb{C}^4$. Apply the quantum Fourier transform to the state
$$|\psi_1\rangle := \frac{1}{2}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle + |2\rangle \otimes |0\rangle + |3\rangle \otimes |1\rangle),$$
and to the state
$$|\psi_2\rangle := \frac{1}{2}(-|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle - |2\rangle \otimes |0\rangle + |3\rangle \otimes |1\rangle)$$
i.e. apply $U_{QFT,4} \otimes I_4$. The quantum Fourier transform on $\mathbb{C}^4$ is given by
$$U_{QFT,4} = \frac{1}{2} \sum_{j=0}^{3} \sum_{k=0}^{3} e^{-i2\pi jk/4} |j\rangle\langle k|.$$
Use your answers to analyze the periodicity of the sequence of states in $|\psi_1\rangle$ and $|\psi_2\rangle$.

2. Let
$$\rho = \begin{pmatrix} a & b \\ \overline{b} & 1-a \end{pmatrix}, \qquad a \in \mathbb{R}, b \in \mathbb{C}, \quad (1-2a)^2 + 4|b|^2 \le 1$$
be an arbitrary density matrix on $\mathbb{C}^2$. Calculate

(a) $\rho_x := \frac{1}{2}\left(\rho + \sigma_x \rho \sigma_x^*\right)$

(b) $\rho_z := \frac{1}{2}\left(\rho + \sigma_z \rho \sigma_z^*\right)$

(c) $\rho_{xz} := \frac{1}{2}\left(\rho_z + \sigma_x \rho_z \sigma_x^*\right)$

(d) $\rho_{zx} := \frac{1}{2}\left(\rho_x + \sigma_z \rho_x \sigma_z^*\right)$

(e) ρ_{xz} in terms of ρ, σ_x, σ_y and σ_z

where
$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$
What is the probability that a measurement, which can distinguish between $(1,0)^T$ and $(0,1)^T$, of the mixed state ρ_{xz} yields the outcome corresponding to $(1,0)^T$?

Discuss the *purification* of the mixed state ρ into the pure state $(1,0)^T$.