UNIVERSITY OF JOHANNESBURG

Applied Mathematics 3B

Assignment #1 **7:30, 26 July 2011**

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1. Let

$$\mathbf{x} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

and

$$\mathbf{y} := \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

be two column vectors.

Calculate $\mathbf{x}^T\mathbf{y}$ and $\mathbf{x}\mathbf{y}^T$.

2. Refer to assignment 1 (2008). Let

$$A := \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}.$$

Determine the eigenvalues λ_1 and λ_2. Determine the corresponding orthonormal eigenvectors $\mathbf{x}_1$ and $\mathbf{x}_2$. Calculate

$$\lambda_1\mathbf{x}_1\mathbf{x}_1^* + \lambda_2\mathbf{x}_2\mathbf{x}_2^*$$

where $\mathbf{x}^*$ denotes the complex conjugate and transpose of $\mathbf{x}$.

3. Find all 2×2 matrices over the real numbers with eigenvalues 1 and -1.

Hint: The characteristic equation for eigenvalues λ of the 2×2 matrix A is

$$\lambda^2 - \mathrm{tr}(A)\lambda + \det(A) = 0.$$

4. Find a 2×2 matrix over the real numbers with only one 1-dimensional eigenspace, i.e. all eigenvectors are linearly dependent. This means that given an eigenvector of the matrix, any vector which is orthogonal (in the 2-dimensional Euclidean space) to the eigenvector is not an eigenvector of the matrix.