TOEGEPASTE WISKUNDE 3B

Semestertoets: 20 Oktober 2009 (Aanvullend)

Tydsduur: 80 minute **Punte:** 30

Instruksies: Beantwoord al die vrae
Alle berekenings moet getoon word
Sakrekenaars mag gebruik word
Alle hoeke word in radiale gemeet
Die voorgeskrewe handboek word toegelaat

Vraag 1

(a) Vind die singulierewaarde dekomposisie van

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

(5)

(b) Bereken die von Neumann entropie van

$$\operatorname{tr}_{\mathcal{H}_A}\left[\frac{1}{2}\begin{pmatrix} p & 0 & 0 & p \\ 0 & 1-p & 0 & 1-p \\ 0 & 0 & 0 & 0 \\ p & 1-p & 0 & 1 \end{pmatrix}\right], \qquad p \in [0,1]$$

op $\mathcal{H}_A \otimes \mathcal{H}_B$ waar $\mathcal{H}_A = \mathcal{H}_B = \mathbb{C}^2$. (5)

(10)

Vraag 2

Laat $x_1, x_2, x_3, x_4 \in \mathbb{C}$. Wys dat $\mathbf{x} = (x_1, x_2, x_3, x_4)^T$ skeibaar op $\mathbb{C}^2 \otimes \mathbb{C}^2$ is as en slegs as

$$\det\begin{pmatrix} x_1 & x_3 \\ x_2 & x_4 \end{pmatrix} = 0.$$

Gebruik hierdie kondisie om te bepaal wanneer

$$\boldsymbol{\psi} := (U_H \otimes U_H)\,\mathbf{x}$$

en

$$\boldsymbol{\phi} := U_{CNOT}\,\mathbf{x}$$

skeibaar is waar

$$I_2 := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad U_H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad U_{CNOT} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

(10)

Vraag 3

Laat

$$\{\, |0\rangle,\, |1\rangle,\, |2\rangle,\, |3\rangle,\, |4\rangle,\, |5\rangle \,\}$$

'n ortonormale basis in $\mathbb{C}^6$ wees. Pas die kwantum Fourier-transformasie toe op die toestand

$$|\psi_1\rangle := \frac{1}{\sqrt{6}}(-|0\rangle + |1\rangle - |2\rangle + |3\rangle - |4\rangle + |5\rangle),$$

en

$$|\psi_2\rangle := \frac{1}{\sqrt{2}}(|0\rangle + |3\rangle).$$

Die kwantum Fourier-transformasie in $\mathbb{C}^6$ is

$$U_{QFT,6} = \frac{1}{\sqrt{6}} \sum_{j=0}^{5} \sum_{k=0}^{5} e^{-i2\pi jk/6} |j\rangle\langle k|.$$

Gebruik u antwoorde om die periodisiteit van die reeks van koëffisiente in $|\psi_1\rangle$ en $|\psi_2\rangle$ te voorspel. **(10)**

EINDE VAN VRAESTEL

APPLIED MATHEMATICS 3B

Semester Test: 20 October 2009 (Supplementary)

Duration: 80 minutes **Marks:** 30

Instructions: Answer all the questions
All calculations must be shown
Pocket calculators are permitted
All angles are measured in radians
The prescribed text book is allowed

Question 1

(a) Find the singular value decomposition of

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

(5)

(b) Calculate the von Neumann entropy of

$$\operatorname{tr}_{\mathcal{H}_A}\left[\frac{1}{2}\begin{pmatrix} p & 0 & 0 & p \\ 0 & 1-p & 0 & 1-p \\ 0 & 0 & 0 & 0 \\ p & 1-p & 0 & 1 \end{pmatrix}\right], \qquad p \in [0,1]$$

on $\mathcal{H}_A \otimes \mathcal{H}_B$ where $\mathcal{H}_A = \mathcal{H}_B = \mathbb{C}^2$. (5)

(10)

Question 2

Let $x_1, x_2, x_3, x_4 \in \mathbb{C}$. Show that $\mathbf{x} = (x_1, x_2, x_3, x_4)^T$ is separable on $\mathbb{C}^2 \otimes \mathbb{C}^2$ if and only if

$$\det\begin{pmatrix} x_1 & x_3 \\ x_2 & x_4 \end{pmatrix} = 0.$$

Use this condition to determine when

$$\boldsymbol{\psi} := (U_H \otimes U_H)\,\mathbf{x}$$

and

$$\boldsymbol{\phi} := U_{CNOT}\,\mathbf{x}$$

are separable where

$$I_2 := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad U_H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad U_{CNOT} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

(10)

Question 3 follows

Question 3

Let

$$\{\,|0\rangle,\, |1\rangle,\, |2\rangle,\, |3\rangle,\, |4\rangle,\, |5\rangle\,\}$$

denote an orthonormal basis in $\mathbb{C}^6$. Apply the quantum Fourier transform to the state

$$|\psi_1\rangle := \frac{1}{\sqrt{6}}(-|0\rangle + |1\rangle - |2\rangle + |3\rangle - |4\rangle + |5\rangle),$$

and

$$|\psi_2\rangle := \frac{1}{\sqrt{2}}(|0\rangle + |3\rangle).$$

The quantum Fourier transform on $\mathbb{C}^6$ is given by

$$U_{QFT,6} = \frac{1}{\sqrt{6}} \sum_{j=0}^{5} \sum_{k=0}^{5} e^{-i2\pi jk/6} |j\rangle\langle k|.$$

Use your answers to predict the periodicity of the sequence of coefficients in $|\psi_1\rangle$ and $|\psi_2\rangle$. **(10)**

END OF QUESTION PAPER