TOEGEPASTE WISKUNDE 3B

Semestertoets: 6 Oktober 2009

Tydsduur: 80 minute **Punte:** 30

Instruksies: Beantwoord al die vrae
Alle berekenings moet getoon word
Sakrekenaars mag gebruik word
Alle hoeke word in radiale gemeet
Die voorgeskrewe handboek word toegelaat

Vraag 1

(a) Vind die singulierewaarde dekomposisie van

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 1 \end{pmatrix}.$$

(5)

(b) Bereken die von Neumann entropie van

$$\mathrm{tr}_{\mathcal{H}_A}\begin{pmatrix} \frac{3}{8} & 0 & \frac{1}{8} & 0 \\ 0 & \frac{1}{8} & 0 & 0 \\ \frac{1}{8} & 0 & \frac{3}{8} & 0 \\ 0 & 0 & 0 & \frac{1}{8} \end{pmatrix}$$

op $\mathcal{H}_A \otimes \mathcal{H}_B$ waar $\mathcal{H}_A = \mathcal{H}_B = \mathbb{C}^2$. (5)

(10)

Vraag 2

Laat $a_1, a_2, b_1, b_2 \in \mathbb{R}$. Bereken

$$\boldsymbol{\psi} := U_{CNOT}(U_H \otimes I_2)\left[\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}\right]$$

waar

$$I_2 := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad U_H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad U_{CNOT} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Bepaal die beperkings op a_1, a_2, b_1, en b_2 sodat $\boldsymbol{\psi}$ skeibaar op $\mathbb{R}^2 \otimes \mathbb{R}^2$ is.

Bepaal die beperkings op a_1, a_2, b_1, en b_2 sodat $\boldsymbol{\psi}$ verstrik op $\mathbb{R}^2 \otimes \mathbb{R}^2$ is. **(10)**

Vraag 3

Laat

$$\{\,|0\rangle,\,|1\rangle,\,|2\rangle,\,|3\rangle\,\}$$

'n ortonormale basis in $\mathbb{C}^4$ wees. Pas die kwantum Fourier-transformasie toe op die toestand

$$|\psi_1\rangle := \frac{1}{\sqrt{4}}(-|0\rangle + |1\rangle - |2\rangle + |3\rangle),$$

en

$$|\psi_2\rangle := \frac{1}{\sqrt{10}}(|0\rangle + 2|1\rangle + |2\rangle + 2|3\rangle),$$

Die kwantum Fourier-transformasie in $\mathbb{C}^4$ is

$$U_{QFT,4} = \frac{1}{\sqrt{4}}\sum_{j=0}^{3}\sum_{k=0}^{3} e^{-i2\pi jk/4}|j\rangle\langle k|.$$

Gebruik u antwoorde om die periodisiteit van die reeks van koëffisiente in $|\psi_1\rangle$ en $|\psi_2\rangle$ te voorspel. **(10)**

EINDE VAN VRAESTEL

APPLIED MATHEMATICS 3B

Semester Test: 6 October 2009

Duration: 80 minutes **Marks:** 30

Instructions: Answer all the questions
All calculations must be shown
Pocket calculators are permitted
All angles are measured in radians
The prescribed text book is allowed

Question 1

(a) Find the singular value decomposition of

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 1 \end{pmatrix}.$$

(5)

(b) Calculate the von Neumann entropy of

$$\mathrm{tr}_{\mathcal{H}_A}\begin{pmatrix} \frac{3}{8} & 0 & \frac{1}{8} & 0 \\ 0 & \frac{1}{8} & 0 & 0 \\ \frac{1}{8} & 0 & \frac{3}{8} & 0 \\ 0 & 0 & 0 & \frac{1}{8} \end{pmatrix}$$

on $\mathcal{H}_A \otimes \mathcal{H}_B$ where $\mathcal{H}_A = \mathcal{H}_B = \mathbb{C}^2$. (5)

(10)

Question 2

Let $a_1, a_2, b_1, b_2 \in \mathbb{R}$. Calculate

$$\boldsymbol{\psi} := U_{CNOT}(U_H \otimes I_2)\left[\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}\right]$$

where

$$I_2 := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad U_H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad U_{CNOT} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Determine the constraints on a_1, a_2, b_1, and b_2 such that $\boldsymbol{\psi}$ is separable on $\mathbb{R}^2 \otimes \mathbb{R}^2$.

Determine the constraints on a_1, a_2, b_1, and b_2 such that $\boldsymbol{\psi}$ is entangled on $\mathbb{R}^2 \otimes \mathbb{R}^2$. **(10)**

Question 3 follows

Question 3

Let

$$\{\,|0\rangle,\,|1\rangle,\,|2\rangle,\,|3\rangle\,\}$$

denote an orthonormal basis in $\mathbb{C}^4$. Apply the quantum Fourier transform to the state

$$|\psi_1\rangle := \frac{1}{\sqrt{4}}(-|0\rangle + |1\rangle - |2\rangle + |3\rangle),$$

and

$$|\psi_2\rangle := \frac{1}{\sqrt{10}}(|0\rangle + 2|1\rangle + |2\rangle + 2|3\rangle),$$

The quantum Fourier transform on $\mathbb{C}^4$ is given by

$$U_{QFT,4} = \frac{1}{\sqrt{4}}\sum_{j=0}^{3}\sum_{k=0}^{3} e^{-i2\pi jk/4}|j\rangle\langle k|.$$

Use your answers to predict the periodicity of the sequence of coefficients in $|\psi_1\rangle$ and $|\psi_2\rangle$. **(10)**

END OF QUESTION PAPER