UNIVERSITEIT VAN JOHANNESBURG

TOEGEPASTE WISKUNDE 3B

Semestertoets: 20 Oktober 2009 (Aanvullend)

Tydsduur: 80 minute **Punte:** 30

Instruksies: Beantwoord al die vrae
Alle berekenings moet getoon word
Sakrekenaars mag gebruik word
Alle hoeke word in radiale gemeet
Die voorgeskrewe handboek word toegelaat

Vraag 1

Laat $a, b, c, d \in \mathbb{C}$. Elke 2×2 matriks H kan in die vorm

$$H = a\sigma_x + b\sigma_y + c\sigma_z + dI_2$$

geskryf word.

(a) Bepaal a, b, c, d sodat H 'n Hermitiese matriks is. (2)

(b) Wys dat

$$H^2 = \mathrm{tr}(H)H - \det(H)I_2, \qquad \mathrm{tr}(H) = 2d, \quad \det(H) = d^2 - c^2 - b^2 - a^2.$$

(2)

(c) Bepaal $\exp(itH)$ vir $\det(H) = 0$. (3)

(d) Bepaal $\exp(itH)$ vir $\mathrm{tr}(H) = 0$. (3)

(10)

Vraag 2

Beskou die Hilbertruimte $\mathbb{C}^2$ en die waarneembaar beskryf deur die matriks

$$\frac{1}{2}\begin{pmatrix} 1 & \cos(2t) \\ \cos(2t) & 1 \end{pmatrix}, \qquad t \in \mathbb{R}.$$

Beskryf die uitkomstes en geassosieerde waarskynlikhede vir die meting van 'n stelsel beskryf deur die digtheidsmatriks

$$\rho_1 := \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

en die meting van 'n stelsel beskryf deur die digtheidmatriks

$$\rho_2 := \frac{1}{2}\begin{pmatrix} 1 & -\cos(2t) \\ -\cos(2t) & 1 \end{pmatrix}.$$

Bereken die verwagte waardes. **(10)**

Vraag 3 volg

Vraag 3

(a) Vind alle 2×2 self-invers unitêre matrikse oor $\mathbb{C}$. (5)

(b) Laat $\{ |0\rangle, |1\rangle \}$ 'n ortonormale basis in $\mathbb{C}^2$ wees. Met ander woorde

$$\langle 0|1\rangle = \langle 1|0\rangle = 0, \qquad \langle 0|0\rangle = \langle 1|1\rangle = 1.$$

Bewys of bewys die teendeel van

$$\left[\frac{1}{\sqrt{2}}(\sigma_a + \sigma_b)\right] \sigma_b \left[\frac{1}{\sqrt{2}}(\sigma_a + \sigma_b)\right] = \sigma_a, \qquad \forall a, b \in \{x, y, z\}, \quad a \neq b.$$

waar

$$\sigma_z := |0\rangle\langle 0| - |1\rangle\langle 1|, \qquad \sigma_x := |0\rangle\langle 1| + |1\rangle\langle 0|, \qquad \sigma_y := -i|0\rangle\langle 1| + i|1\rangle\langle 0|.$$

(5)

(10)

EINDE VAN VRAESTEL

APPLIED MATHEMATICS 3B

Semester Test: 20 October 2009 (Supplementary)

Duration: 80 minutes **Marks:** 30

Instructions: Answer all the questions
All calculations must be shown
Pocket calculators are permitted
All angles are measured in radians
The prescribed text book is allowed

Question 1

Let $a, b, c, d \in \mathbb{C}$. Every 2×2 matrix H can be written in the form

$$H = a\sigma_x + b\sigma_y + c\sigma_z + dI_2.$$

(a) Determine a, b, c, d such that H is Hermitian. (2)

(b) Show that

$$H^2 = \operatorname{tr}(H)H - \det(H)I_2, \qquad \operatorname{tr}(H) = 2d, \quad \det(H) = d^2 - c^2 - b^2 - a^2.$$

(2)

(c) Determine $\exp(itH)$ for $\det(H) = 0$. (3)

(d) Determine $\exp(itH)$ for $\operatorname{tr}(H) = 0$. (3)

(10)

Question 2

Consider the Hilbert space $\mathbb{C}^2$ and the observable described by the matrix

$$\frac{1}{2}\begin{pmatrix} 1 & \cos(2t) \\ \cos(2t) & 1 \end{pmatrix}, \qquad t \in \mathbb{R}.$$

Describe the measurement outcomes and associated probabilities when observing a system described by the density matrix

$$\rho_1 := \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

and when observing a system described by the density matrix

$$\rho_2 := \frac{1}{2}\begin{pmatrix} 1 & -\cos(2t) \\ -\cos(2t) & 1 \end{pmatrix}.$$

Calculate the expectation values. **(10)**

Question 3 follows

Question 3

(a) Find all 2×2 self inverse unitary matrices over $\mathbb{C}$. (5)

(b) Let $\{\,|0\rangle, |1\rangle\,\}$ denote an orthonormal basis in $\mathbb{C}^2$. In other words

$$\langle 0|1\rangle = \langle 1|0\rangle = 0, \qquad \langle 0|0\rangle = \langle 1|1\rangle = 1.$$

Prove or disprove the equality

$$\left[\frac{1}{\sqrt{2}}(\sigma_a + \sigma_b)\right] \sigma_b \left[\frac{1}{\sqrt{2}}(\sigma_a + \sigma_b)\right] = \sigma_a, \qquad \forall a, b \in \{x, y, z\}, \quad a \neq b.$$

where

$$\sigma_z := |0\rangle\langle 0| - |1\rangle\langle 1|, \qquad \sigma_x := |0\rangle\langle 1| + |1\rangle\langle 0|, \qquad \sigma_y := -i|0\rangle\langle 1| + i|1\rangle\langle 0|.$$

(5)

(10)

END OF QUESTION PAPER