UNIVERSITEIT VAN JOHANNESBURG

TOEGEPASTE WISKUNDE 3B

Semestertoets: 25 Augustus 2009

Tydsduur: 80 minute **Punte:** 30

Instruksies: Beantwoord al die vrae
Alle berekenings moet getoon word
Sakrekenaars mag gebruik word
Alle hoeke word in radiale gemeet
Die voorgeskrewe handboek word toegelaat

Vraag 1

Los die differensiaalvergelyking (beginwaarde probleem)

$$\frac{d\psi}{dt} = i\omega \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \psi$$

op vir $\psi(t) \in \mathbb{C}^4$. Vind $\psi(t = \frac{\pi}{2\omega})$. Beskou die gevalle

$$\psi(t=0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \psi(t=0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

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Vraag 2

Beskou die Hilbertruimte $\mathbb{C}^2$ en die waarneembaar beskryf deur die matriks

$$\frac{1}{2}\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}.$$

Beskryf die uitkomstes en geassosieerde waarskynlikhede vir die meting van 'n stelsel beskryf deur die digtheidsmatriks

$$\rho_1 := \cos^2\theta \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}^* + \sin^2\theta \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}^*, \qquad \theta \in \mathbb{R}$$

en die meting van 'n stelsel beskryf deur die digtheidmatriks

$$\rho_2 := \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}^*, \qquad \theta \in \mathbb{R}.$$

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Vraag 3 volg

Vraag 3

Laat $\{\,|0\rangle,\,|1\rangle\,\}$ 'n ortonormale basis in $\mathbb{C}^2$ wees. Met ander woorde

$$\langle 0|1\rangle = \langle 1|0\rangle = 0, \qquad \langle 0|0\rangle = \langle 1|1\rangle = 1.$$

Bewys of bewys die teendeel van

$$(U_H \otimes I_2)\Big(|0\rangle\langle 0| \otimes I_2 + |1\rangle\langle 1| \otimes \sigma_z\Big)(U_H \otimes I_2) = (U_H \otimes I_2)\Big(I_2 \otimes |0\rangle\langle 0| + \sigma_z \otimes |1\rangle\langle 1|\Big)(U_H \otimes I_2)$$

waar

$$I_2 := |0\rangle\langle 0| + |1\rangle\langle 1|, \qquad \sigma_z := |0\rangle\langle 0| - |1\rangle\langle 1|, \qquad \sigma_x := |0\rangle\langle 1| + |1\rangle\langle 0|$$

en

$$\begin{aligned} U_H &:= \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)\langle 0| + \frac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big)\langle 1| \\ &= \frac{1}{\sqrt{2}}|0\rangle\big(\langle 0| + \langle 1|\big) + \frac{1}{\sqrt{2}}|1\rangle\big(\langle 0| - \langle 1|\big). \end{aligned}$$

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EINDE VAN VRAESTEL

APPLIED MATHEMATICS 3B

Semester Test: 25 August 2009

Duration: 80 minutes **Marks:** 30

Instructions: Answer all the questions
All calculations must be shown
Pocket calculators are permitted
All angles are measured in radians
The prescribed text book is allowed

Question 1

Solve the differential equation (initial value problem)

$$\frac{d\psi}{dt} = i\omega \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \psi$$

for $\psi(t) \in \mathbb{C}^4$. Find $\psi(t = \frac{\pi}{2\omega})$. Consider the cases

$$\psi(t=0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \psi(t=0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

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Question 2

Consider the Hilbert space $\mathbb{C}^2$ and the observable described by the matrix

$$\frac{1}{2}\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}.$$

Describe the measurement outcomes and associated probabilities when observing a system described by the density matrix

$$\rho_1 := \cos^2\theta \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}^* + \sin^2\theta \begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}^*, \qquad \theta \in \mathbb{R}$$

and when observing a system described by the density matrix

$$\rho_2 := \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}^*, \qquad \theta \in \mathbb{R}.$$

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Question 3 follows

Question 3

Let $\{ |0\rangle, |1\rangle \}$ denote an orthonormal basis in $\mathbb{C}^2$. In other words

$$\langle 0|1\rangle = \langle 1|0\rangle = 0, \qquad \langle 0|0\rangle = \langle 1|1\rangle = 1.$$

Prove or disprove the equality

$$(U_H \otimes I_2)\Big(|0\rangle\langle 0| \otimes I_2 + |1\rangle\langle 1| \otimes \sigma_z\Big)(U_H \otimes I_2) = (U_H \otimes I_2)\Big(I_2 \otimes |0\rangle\langle 0| + \sigma_z \otimes |1\rangle\langle 1|\Big)(U_H \otimes I_2)$$

where

$$I_2 := |0\rangle\langle 0| + |1\rangle\langle 1|, \qquad \sigma_z := |0\rangle\langle 0| - |1\rangle\langle 1|, \qquad \sigma_x := |0\rangle\langle 1| + |1\rangle\langle 0|$$

and

$$\begin{aligned} U_H &:= \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)\langle 0| + \frac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big)\langle 1| \\ &= \frac{1}{\sqrt{2}}|0\rangle\big(\langle 0| + \langle 1|\big) + \frac{1}{\sqrt{2}}|1\rangle\big(\langle 0| - \langle 1|\big). \end{aligned}$$

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END OF QUESTION PAPER