UNIVERSITEIT VAN JOHANNESBURG

FAKULTEIT NATUURWETENSKAP

DEPARTEMENT TOEGEPASTE WISKUNDE

MODULE **APM3B10**
KWANTUMBEREKENINGE

KAMPUS **APK**

EKSAMEN **NOVEMBER 2009**

DATUM: 3/11/2009 **SESSION:** 09:00 – 12:00

ASSESSOR DR. Y. HARDY

EKSTERNE MODERATOR PROF. R. DE MELLO KOCH

TYDSDUUR: 3 UUR **PUNTE:** 40

AANTAL BLADSYE: 3 BLADSYE

INSTRUKSIES: BEANTWOORD AL DIE VRAE

ALLE BEREKENINGS MOET GETOON WORD

SAKREKENAARS MAG GEBRUIK WORD

ALLE HOEKE WORD IN RADIALE GEMEET

DIE VOORGESKREWE HANDBOEK WORD TOEGELAAT

See reverse for English

VRAAG 1

Beskou die Hilbertruimte $\mathbb{C}^4$ en die waarneembaar beskryf deur die matriks

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Beskryf die uitkomstes en geassosieerde waarskynlikhede vir die meting van 'n stelsel beskryf deur die digtheidsmatriks

$$\rho := \frac{1}{4}\begin{pmatrix} \sin\theta \\ \cos\theta \\ \cos\theta \\ \sin\theta \end{pmatrix}\begin{pmatrix} \sin\theta \\ \cos\theta \\ \cos\theta \\ \sin\theta \end{pmatrix}^* + \frac{1}{4}\begin{pmatrix} \cos\theta \\ \sin\theta \\ \sin\theta \\ \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta \\ \sin\theta \\ \sin\theta \\ \cos\theta \end{pmatrix}^*, \qquad \theta \in \mathbb{R}.$$

Bereken die verwagte waarde. **(10)**

VRAAG 2

(a) Bereken die von Neumann entropie van

$$\operatorname{tr}_{\mathcal{H}_A}\begin{pmatrix} \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix}$$

op $\mathcal{H}_A \otimes \mathcal{H}_B$ waar $\mathcal{H}_A = \mathcal{H}_B = \mathbb{C}^2$. (5)

(b) Laat $\mathbf{x} = (x_1, x_2, x_3, x_4)^T \in \mathbb{C}^2 \otimes \mathbb{C}^2$ en definieer

$$\widetilde{\mathbf{x}} := [(i\sigma_y) \otimes (i\sigma_y)]\,\mathbf{x}.$$

Wys dat $\mathbf{x}$ skeibaar op $\mathbb{C}^2 \otimes \mathbb{C}^2$ is as en slegs as $\mathbf{x}^T\widetilde{\mathbf{x}} = 0$.

Laat $\mathbf{a}, \mathbf{b} \in \mathbb{C}^2 \otimes \mathbb{C}^2$ waar $\mathbf{a}$ verstrik op $\mathbb{C}^2 \otimes \mathbb{C}^2$ is. Vind $\alpha \in \mathbb{C}$ sodat

$$\alpha\mathbf{a} + \mathbf{b}$$

skeibaar op $\mathbb{C}^2 \otimes \mathbb{C}^2$ is, of bewys dat so 'n α nie bestaan nie. (5)

(10)

VRAAG 3

Laat

$$\{\, |0\rangle,\ |1\rangle,\ |2\rangle,\ |3\rangle \,\}$$

'n ortonormale basis in $\mathbb{C}^4$ wees. Pas die kwantum Fourier-transformasie op $\mathbb{C}^4$ toe op die toestand

$$|\psi_1\rangle := \frac{1}{2}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |2\rangle + |2\rangle \otimes |1\rangle + |3\rangle \otimes |2\rangle),$$

en op die toestand

$$|\psi_2\rangle := \frac{1}{2}(|0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle + |2\rangle \otimes |1\rangle - |3\rangle \otimes |0\rangle)$$

d.w.s. pas $U_{QFT,4} \otimes I_4$ toe. Die kwantum Fourier-transformasie in $\mathbb{C}^4$ is

$$U_{QFT,4} = \frac{1}{2}\sum_{j=0}^{3}\sum_{k=0}^{3} e^{-i2\pi jk/4}|j\rangle\langle k|.$$

Gebruik u antwoorde om die periodisiteit van die ry van toestande in $|\psi_1\rangle$ en $|\psi_2\rangle$ te voorspel.

(10)

VRAAG 4

Laat $U(t)$ en H (tyd onafhanklik) twee $n \times n$ matrikse oor $\mathbb{C}$ wees. Neem aan dat

$$U(t) = e^{-iHt}$$

waar $t \in \mathbb{R}$ en $H = H^*$.

(a) Wys dat $U(t)$ 'n unitêre matriks is. (2)

(b) Wys dat

$$H = iU(t)^* \frac{d}{dt} U(t).$$

(3)

(c) Laat

$$U(t) = \begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix}.$$

Vind H. (5)

(10)

EINDE VAN VRAESTEL

FACULTY OF SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

MODULE **APM3B10**
QUANTUM COMPUTING

CAMPUS **APK**

EXAM **NOVEMBER 2009**

DATE: 3/11/2009 **SESSION:** 09:00 – 12:00

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DURATION: 3 HOURS **MARKS:** 40

NUMBER OF PAGES: 3 PAGES

INSTRUCTIONS: ANSWER ALL THE QUESTIONS

ALL CALCULATIONS MUST BE SHOWN

POCKET CALCULATORS ARE PERMITTED

ALL ANGLES ARE MEASURED IN RADIANS

THE PRESCRIBED TEXT BOOK IS ALLOWED

Sien keersy vir Afrikaans

QUESTION 1

Consider the Hilbert space $\mathbb{C}^4$ and the observable described by the matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Describe the measurement outcomes and associated probabilities when observing a system described by the density matrix

$$\rho := \frac{1}{4}\begin{pmatrix} \sin\theta \\ \cos\theta \\ \cos\theta \\ \sin\theta \end{pmatrix}\begin{pmatrix} \sin\theta \\ \cos\theta \\ \cos\theta \\ \sin\theta \end{pmatrix}^* + \frac{1}{4}\begin{pmatrix} \cos\theta \\ \sin\theta \\ \sin\theta \\ \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta \\ \sin\theta \\ \sin\theta \\ \cos\theta \end{pmatrix}^*, \qquad \theta \in \mathbb{R}.$$

Calculate the expectation value. **(10)**

QUESTION 2

(a) Calculate the von Neumann entropy of

$$\mathrm{tr}_{\mathcal{H}_A}\begin{pmatrix} \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix}$$

on $\mathcal{H}_A \otimes \mathcal{H}_B$ where $\mathcal{H}_A = \mathcal{H}_B = \mathbb{C}^2$. (5)

(b) Let $\mathbf{x} = (x_1, x_2, x_3, x_4)^T \in \mathbb{C}^2 \otimes \mathbb{C}^2$ and define

$$\widetilde{\mathbf{x}} := [(i\sigma_y) \otimes (i\sigma_y)]\,\mathbf{x}.$$

Show that $\mathbf{x}$ is separable on $\mathbb{C}^2 \otimes \mathbb{C}^2$ if and only if $\mathbf{x}^T\widetilde{\mathbf{x}} = 0$.

Let $\mathbf{a}, \mathbf{b} \in \mathbb{C}^2 \otimes \mathbb{C}^2$ where $\mathbf{a}$ is entangled on $\mathbb{C}^2 \otimes \mathbb{C}^2$. Find $\alpha \in \mathbb{C}$ such that

$$\alpha\mathbf{a} + \mathbf{b}$$

is separable on $\mathbb{C}^2 \otimes \mathbb{C}^2$, or prove that no such α exists. (5)

(10)

QUESTION 3

Let

$$\{\, |0\rangle,\, |1\rangle,\, |2\rangle,\, |3\rangle \,\}$$

denote an orthonormal basis in $\mathbb{C}^4$. Apply the quantum Fourier transform on $\mathbb{C}^4$ to the state

$$|\psi_1\rangle := \frac{1}{2}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |2\rangle + |2\rangle \otimes |1\rangle + |3\rangle \otimes |2\rangle),$$

and to the state

$$|\psi_2\rangle := \frac{1}{2}(|0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle + |2\rangle \otimes |1\rangle - |3\rangle \otimes |0\rangle)$$

i.e. apply $U_{QFT,4} \otimes I_4$. The quantum Fourier transform on $\mathbb{C}^4$ is given by

$$U_{QFT,4} = \frac{1}{2}\sum_{j=0}^{3}\sum_{k=0}^{3} e^{-i2\pi jk/4}|j\rangle\langle k|.$$

Use your answers to predict the periodicity of the sequence of states in $|\psi_1\rangle$ and $|\psi_2\rangle$. **(10)**

QUESTION 4

Let $U(t)$ and H (time independent) be $n \times n$ matrices over $\mathbb{C}$. Assume that

$$U(t) = e^{-iHt}$$

where $t \in \mathbb{R}$ and $H = H^*$.

(a) Show that $U(t)$ is unitary. (2)

(b) Show that

$$H = iU(t)^* \frac{d}{dt} U(t).$$

(3)

(c) Let

$$U(t) = \begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix}.$$

Find H. (5)

(10)

END OF QUESTION PAPER