Toegepaste Wiskunde 3B

Taak #9 **7:30, 29 September 2009**

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1. Laat

$$\boldsymbol{\phi}^+ := \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}, \qquad \boldsymbol{\phi}^- := \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}, \qquad \boldsymbol{\psi}^+ := \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\\0\end{pmatrix}, \qquad \boldsymbol{\psi}^- := \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\\0\end{pmatrix}.$$

Watter van die toestande is skeibaar? D.w.s. vir watter toestande

$$\mathbf{x} \in \{\boldsymbol{\phi}^+, \boldsymbol{\phi}^-, \boldsymbol{\psi}^+, \boldsymbol{\psi}^-\}$$

bestaan $(a_1, a_2)^T, (b_1, b_2)^T \in \mathbb{C}^2$ met

$$(a_1, a_2)^T \otimes (b_1, b_2)^T = \mathbf{x}?$$

2. Verwys na hoofstuk 9, probleem 4 in die handboek

Problems and Solutions in Quantum Computing and Quantum Information, 2de uitgawe.

Laat $\mathcal{H}_A = \mathcal{H}_B = \mathbb{C}^2$ en

$$\boldsymbol{\phi}^+ := \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}, \qquad \boldsymbol{\phi}^- := \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}, \qquad \boldsymbol{\psi}^+ := \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\\0\end{pmatrix}, \qquad \boldsymbol{\psi}^- := \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\\0\end{pmatrix}$$

in die Hilbertruimte $\mathcal{H}_A \otimes \mathcal{H}_B = \mathbb{C}^4$. Laat

$$\rho_1 := \boldsymbol{\phi}^+(\boldsymbol{\phi}^+)^*, \quad \rho_2 := \frac{1}{2}\boldsymbol{\phi}^+(\boldsymbol{\phi}^+)^* + \frac{1}{2}\boldsymbol{\phi}^-(\boldsymbol{\phi}^-)^*$$

en

$$\rho_3 := \frac{1}{4}\boldsymbol{\phi}^+(\boldsymbol{\phi}^+)^* + \frac{1}{4}\boldsymbol{\phi}^-(\boldsymbol{\phi}^-)^* + \frac{1}{4}\boldsymbol{\psi}^+(\boldsymbol{\psi}^+)^* + \frac{1}{4}\boldsymbol{\psi}^-(\boldsymbol{\psi}^-)^*.$$

Bereken die von Neumann entropie van

$$\rho_{1,A} = \mathrm{tr}_{\mathcal{H}_A}\rho_1, \quad \rho_{2,A} = \mathrm{tr}_{\mathcal{H}_A}\rho_2, \quad \rho_{3,A} = \mathrm{tr}_{\mathcal{H}_A}\rho_3.$$

Applied Mathematics 3B

Assignment #9 **7:30, 29 September 2009**

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1. Let

$$\boldsymbol{\phi}^+ := \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}, \qquad \boldsymbol{\phi}^- := \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}, \qquad \boldsymbol{\psi}^+ := \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\\0\end{pmatrix}, \qquad \boldsymbol{\psi}^- := \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\\0\end{pmatrix}.$$

Which of these states are separable? I.e. for which states

$$\mathbf{x} \in \{\boldsymbol{\phi}^+, \boldsymbol{\phi}^-, \boldsymbol{\psi}^+, \boldsymbol{\psi}^-\}$$

do $(a_1, a_2)^T, (b_1, b_2)^T \in \mathbb{C}^2$ exist such that

$$(a_1, a_2)^T \otimes (b_1, b_2)^T = \mathbf{x}?$$

2. Refer to chapter 9, problem 4 in the textbook

Problems and Solutions in Quantum Computing and Quantum Information, 2nd edition.

Let $\mathcal{H}_A = \mathcal{H}_B = \mathbb{C}^2$ and

$$\boldsymbol{\phi}^+ := \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}, \qquad \boldsymbol{\phi}^- := \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}, \qquad \boldsymbol{\psi}^+ := \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\\0\end{pmatrix}, \qquad \boldsymbol{\psi}^- := \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\\0\end{pmatrix}$$

in the Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B = \mathbb{C}^4$. Let

$$\rho_1 := \boldsymbol{\phi}^+(\boldsymbol{\phi}^+)^*, \quad \rho_2 := \frac{1}{2}\boldsymbol{\phi}^+(\boldsymbol{\phi}^+)^* + \frac{1}{2}\boldsymbol{\phi}^-(\boldsymbol{\phi}^-)^*$$

and

$$\rho_3 := \frac{1}{4}\boldsymbol{\phi}^+(\boldsymbol{\phi}^+)^* + \frac{1}{4}\boldsymbol{\phi}^-(\boldsymbol{\phi}^-)^* + \frac{1}{4}\boldsymbol{\psi}^+(\boldsymbol{\psi}^+)^* + \frac{1}{4}\boldsymbol{\psi}^-(\boldsymbol{\psi}^-)^*.$$

Calculate the von Neumann entropies of

$$\rho_{1,A} = \mathrm{tr}_{\mathcal{H}_A}\rho_1, \quad \rho_{2,A} = \mathrm{tr}_{\mathcal{H}_A}\rho_2, \quad \rho_{3,A} = \mathrm{tr}_{\mathcal{H}_A}\rho_3.$$