UNIVERSITEIT VAN JOHANNESBURG

Toegepaste Wiskunde 3B

Taak #8 **7:30, 22 September 2009**

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1. Laat

$$\{\,|0\rangle,\,|1\rangle,\,|2\rangle,\,|3\rangle\,\}$$

'n ortonormale basis in $\mathbb{C}^4$ wees. Pas die kwantum Fourier-transformasie toe op die toestand

$$|\psi_1\rangle := \frac{1}{2}(|0\rangle\otimes|0\rangle + |1\rangle\otimes|1\rangle + |2\rangle\otimes|0\rangle + |3\rangle\otimes|1\rangle),$$

en op die toestand

$$|\psi_2\rangle := \frac{1}{2}(|0\rangle\otimes|0\rangle + |1\rangle\otimes|1\rangle - |2\rangle\otimes|0\rangle - |3\rangle\otimes|1\rangle)$$

d.w.s. pas $U_{QFT,4}\otimes I_4$ toe. Die kwantum Fourier-transformasie in $\mathbb{C}^4$ is

$$U_{QFT,4} = \frac{1}{2}\sum_{j=0}^{3}\sum_{k=0}^{3} e^{-i2\pi jk/4}|j\rangle\langle k|.$$

Gebruik u antwoorde om die periodisiteit van die ry van toestande in $|\psi_1\rangle$ en $|\psi_2\rangle$ te analiseer.

2. Laat $\{\,|0\rangle,\,|1\rangle\,\}$ 'n ortonormale basis in $\mathbb{C}^2$ wees. Met ander woorde

$$\langle 0|1\rangle = \langle 1|0\rangle = 0, \qquad \langle 0|0\rangle = \langle 1|1\rangle = 1.$$

$$|\psi\rangle := |0\rangle\otimes|1\rangle - |1\rangle\otimes|0\rangle.$$

Bereken die gedeeltelike spoor

$$\sum_{j=0}^{1}(\langle j|\otimes I_2)\,|\psi\rangle\langle\psi|\,(|j\rangle\otimes I_2).$$

en

$$\sum_{j=0}^{1}(I_2\otimes\langle j|)\,|\psi\rangle\langle\psi|\,(I_2\otimes|j\rangle).$$

Applied Mathematics 3B

Assignment #8 **7:30, 22 September 2009**

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1. Let

$$\{\,|0\rangle,\,|1\rangle,\,|2\rangle,\,|3\rangle\,\}$$

denote an orthonormal basis in $\mathbb{C}^4$. Apply the quantum Fourier transform to the state

$$|\psi_1\rangle := \frac{1}{2}(|0\rangle\otimes|0\rangle + |1\rangle\otimes|1\rangle + |2\rangle\otimes|0\rangle + |3\rangle\otimes|1\rangle),$$

and to the state

$$|\psi_2\rangle := \frac{1}{2}(|0\rangle\otimes|0\rangle + |1\rangle\otimes|1\rangle - |2\rangle\otimes|0\rangle - |3\rangle\otimes|1\rangle)$$

i.e. apply $U_{QFT,4}\otimes I_4$. The quantum Fourier transform on $\mathbb{C}^4$ is given by

$$U_{QFT,4} = \frac{1}{2}\sum_{j=0}^{3}\sum_{k=0}^{3} e^{-i2\pi jk/4}|j\rangle\langle k|.$$

Use your answers to analyze the periodicity of the sequence of states in $|\psi_1\rangle$ and $|\psi_2\rangle$.

2. Let $\{\,|0\rangle,\,|1\rangle\,\}$ denote an orthonormal basis in $\mathbb{C}^2$. In other words

$$\langle 0|1\rangle = \langle 1|0\rangle = 0, \qquad \langle 0|0\rangle = \langle 1|1\rangle = 1.$$

$$|\psi\rangle := |0\rangle\otimes|1\rangle - |1\rangle\otimes|0\rangle.$$

Calculate the partial traces

$$\sum_{j=0}^{1}(\langle j|\otimes I_2)\,|\psi\rangle\langle\psi|\,(|j\rangle\otimes I_2).$$

and

$$\sum_{j=0}^{1}(I_2\otimes\langle j|)\,|\psi\rangle\langle\psi|\,(I_2\otimes|j\rangle).$$
