UNIVERSITEIT VAN JOHANNESBURG

Toegepaste Wiskunde 3B

Taak #4 **7:30, 11 Augustus 2009**

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1. Beskou die Hilbertruimte $\mathbb{C}^3$. Wys dat die matriks

$$\sigma_x \otimes \sigma_x, \qquad \sigma_x := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

'n waarneembare kwantiteit beskryf. Beskryf die uitkomstes en geassosieerde waarskynlikhede vir die meting van 'n stelsel beskryf deur die toestand

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Is die eigetoestande van $\sigma_x \otimes \sigma_x$ skeibaar?

2. Beskou die Hilbertruimte $\mathbb{C}^2$ en die waarneembaar beskryf deur die matriks

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Beskryf die uitkomstes en geassosieerde waarskynlikhede vir die meting van 'n stelsel beskryf deur die toestand

$$\rho_1 := \mathbf{x}_1 \mathbf{x}_1^*, \qquad \mathbf{x}_1 := \frac{e^{i\theta}}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad \theta \in \mathbb{R}.$$

Beskryf die uitkomstes en geassosieerde waarskynlikhede vir die meting van 'n stelsel beskryf deur die toestand

$$\rho_2 := \mathbf{x}_2 \mathbf{x}_2^*, \qquad \mathbf{x}_2 := \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i\theta} \end{pmatrix}, \qquad \theta \in \mathbb{R}.$$

Bereken die verwagte waarde vir elke geval.

3. Laat $\mathbf{a}, \mathbf{b} \in \mathbb{C}^2$. Los

$$\mathbf{a} \otimes \mathbf{b} = \mathbf{b} \otimes \mathbf{a}$$

op vir $\mathbf{a}$ en $\mathbf{b}$. **Wenk:** Een benadering is om te wys dat

$$\mathbf{a} \otimes \mathbf{b} - \mathbf{b} \otimes \mathbf{a} = \mathbf{0} \quad \Leftrightarrow \quad \det \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} = 0.$$

UNIVERSITY OF JOHANNESBURG

Applied Mathematics 3B

Assignment #4 **7:30, 11 August 2009**

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1. Consider the Hilbert space $\mathbb{C}^4$. Show that the matrix

$$\sigma_x \otimes \sigma_x, \qquad \sigma_x := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

describes an observable. Describe the measurement outcomes and associated probabilities when observing (performing the measurement on) a system described by the state

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Are the eigenstates of $\sigma_x \otimes \sigma_x$ separable?

2. Consider the Hilbert space $\mathbb{C}^2$ and the observable described by the matrix

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Describe the measurement outcomes and associated probabilities when observing a system described by the density matrix

$$\rho_1 := \mathbf{x}_1 \mathbf{x}_1^*, \qquad \mathbf{x}_1 := \frac{e^{i\theta}}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad \theta \in \mathbb{R}.$$

Describe the measurement outcomes and associated probabilities when observing a system described by the state

$$\rho_2 := \mathbf{x}_2 \mathbf{x}_2^*, \qquad \mathbf{x}_2 := \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i\theta} \end{pmatrix}, \qquad \theta \in \mathbb{R}.$$

In each case calculate the expectation value.

3. Let $\mathbf{a}, \mathbf{b} \in \mathbb{C}^2$. Solve

$$\mathbf{a} \otimes \mathbf{b} = \mathbf{b} \otimes \mathbf{a}$$

for $\mathbf{a}$ and $\mathbf{b}$. **Hint:** One approach is to show that

$$\mathbf{a} \otimes \mathbf{b} - \mathbf{b} \otimes \mathbf{a} = \mathbf{0} \quad \Leftrightarrow \quad \det \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} = 0.$$
