Toegepaste Wiskunde 3B

Taak #2 **7:30, 28 Julie 2009**

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1. Wys dat

$$\left\{ \mathbf{x}_1 = \begin{pmatrix} \frac{1}{2} \\ \sqrt{\frac{3}{8}} \\ \sqrt{\frac{3}{8}} \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} -\frac{\sqrt{3}}{2} \\ \sqrt{\frac{1}{8}} \\ \sqrt{\frac{1}{8}} \end{pmatrix}, \mathbf{x}_3 = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}, \right\}$$

'n ortonormale basis in $\mathbb{C}^3$ is.

Bereken

$$\mathbf{x}_1\mathbf{x}_1^* + \mathbf{x}_2\mathbf{x}_2^* + \mathbf{x}_3\mathbf{x}_3^*.$$

2. Wys dat

$$\left\{ \mathbf{x}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} \cos\theta \\ 0 \\ \sin\theta \end{pmatrix}, \mathbf{x}_3 = \begin{pmatrix} \sin\theta \\ 0 \\ -\cos\theta \end{pmatrix}, \right\} \qquad \theta \in \mathbb{R}$$

'n ortonormale basis in $\mathbb{C}^3$ is.

Bereken

$$\mathbf{x}_1\mathbf{x}_1^* + \mathbf{x}_2\mathbf{x}_2^* + \mathbf{x}_3\mathbf{x}_3^*.$$

3. Laat

$$A := \hbar\omega \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

Bepaal die eiewaardes λ_1 en λ_2. Bepaal die ooreenstemmende genormaliseerde eievektore $\mathbf{x}_1$ en $\mathbf{x}_2$. Bereken

$$e^{-it\lambda_1/\hbar}\mathbf{x}_1\mathbf{x}_1^* + e^{-it\lambda_2/\hbar}\mathbf{x}_2\mathbf{x}_2^*.$$

4. Los die differensiaalvergelyking

$$i\hbar\frac{d\boldsymbol{\psi}}{dt} = \left[\hbar\omega \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\right]\boldsymbol{\psi}, \qquad \boldsymbol{\psi}(t), \boldsymbol{\psi}(0) \in \mathbb{C}^2.$$

op vir $\boldsymbol{\psi}(t)$ in terme van $\boldsymbol{\psi}(0)$.

UNIVERSITY OF JOHANNESBURG

Applied Mathematics 3B

Assignment #2 **7:30, 28 July 2009**

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1. Show that

$$\left\{ \mathbf{x}_1 = \begin{pmatrix} \frac{1}{2} \\ \sqrt{\frac{3}{8}} \\ \sqrt{\frac{3}{8}} \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} -\frac{\sqrt{3}}{2} \\ \sqrt{\frac{1}{8}} \\ \sqrt{\frac{1}{8}} \end{pmatrix}, \mathbf{x}_3 = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}, \right\}$$

is an orthonormal basis in $\mathbb{C}^3$.

Calculate

$$\mathbf{x}_1\mathbf{x}_1^* + \mathbf{x}_2\mathbf{x}_2^* + \mathbf{x}_3\mathbf{x}_3^*.$$

2. Show that

$$\left\{ \mathbf{x}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} \cos\theta \\ 0 \\ \sin\theta \end{pmatrix}, \mathbf{x}_3 = \begin{pmatrix} \sin\theta \\ 0 \\ -\cos\theta \end{pmatrix}, \right\} \qquad \theta \in \mathbb{R}$$

is an orthonormal basis in $\mathbb{C}^3$.

Calculate

$$\mathbf{x}_1\mathbf{x}_1^* + \mathbf{x}_2\mathbf{x}_2^* + \mathbf{x}_3\mathbf{x}_3^*.$$

3. Let

$$A := \hbar\omega \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

Determine the eigenvalues λ_1 and λ_2. Determine the corresponding orthonormal eigenvectors $\mathbf{x}_1$ and $\mathbf{x}_2$. Calculate

$$e^{-it\lambda_1/\hbar}\mathbf{x}_1\mathbf{x}_1^* + e^{-it\lambda_2/\hbar}\mathbf{x}_2\mathbf{x}_2^*.$$

4. Solve the differential equation

$$i\hbar\frac{d\boldsymbol{\psi}}{dt} = \left[\hbar\omega \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\right]\boldsymbol{\psi}, \qquad \boldsymbol{\psi}(t), \boldsymbol{\psi}(0) \in \mathbb{C}^2.$$

for $\boldsymbol{\psi}(t)$ in terms of $\boldsymbol{\psi}(0)$.