Toegepaste Wiskunde 3B

Taak #10 **7:30, 13 Oktober 2009**

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1. Vind (die posisie en waardes van) die ekstrema van

$$-x\log_2 x - (1-x)\log_2(1-x), \qquad x \in [0,1].$$

2. Verwys na hoofstuk 9, probleem 2 in die handboek

Problems and Solutions in Quantum Computing and Quantum Information, 2de uitgawe.

Vind $\mathbf{x} \in \mathbb{C}^4$ sodat

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} + \mathbf{x}$$

skeibaar op $\mathbb{C}^2 \otimes \mathbb{C}^2$ is. Is $\mathbf{x}$ verstrik?

3. Laat $|\psi\rangle \in \mathbb{C}^n$ genormaliseerd wees. Die getrouheid van $|\psi\rangle$ en die digtheidsmatriks ρ is

$$F(\rho, |\psi\rangle) := \sqrt{\langle\psi|\rho|\psi\rangle}.$$

Laat X_1, X_2, ..., X_m $n \times n$ unitêre matrikse wees en laat

$$\rho := p_0|\psi\rangle\langle\psi| + \sum_{j=1}^{m} p_j\, X_j|\psi\rangle\langle\psi|X_j^*, \qquad p_0, p_1, \ldots, p_m \in [0,1],\ \sum_{j=0}^{m} p_m = 1.$$

Wys dat

$$F(\rho, |\psi\rangle) = \sqrt{p_0 + \sum_{j=1}^{m} p_j |\langle\psi|X_j|\psi\rangle|^2}.$$

Laat $\{\,|0\rangle, |1\rangle, \}$ 'n ortonormale basis in $\mathbb{C}^2$ wees.

Bereken die minimum getrouheid vir $m = 1$, $n = 2$, en $|\psi\rangle = |0\rangle$ and $X_1 = \sigma_x = U_{NOT}$.

Bereken die minimum getrouheid vir $m = 1$, $n = 2$, $p_0 = p_1 = \frac{1}{2}$,

$$|\psi\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle, \qquad \theta \in [0, 2\pi]$$

en $X_1 = \sigma_x = U_{NOT}$.

Bereken die minimum getrouheid vir $m = 1$, $n = 8$, $p_0 = p_1 = \frac{1}{2}$,

$$|\psi\rangle = \cos\theta\,|0\rangle \otimes |0\rangle \otimes |0\rangle + \sin\theta\,|1\rangle \otimes |1\rangle \otimes |1\rangle, \qquad \theta \in [0, 2\pi]$$

en $X_1 = I_2 \otimes I_2 \otimes \sigma_x$.

Applied Mathematics 3B

Assignment #10 **7:30, 13 October 2009**

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1. Find the (locations and values of the) extrema of

$$-x \log_2 x - (1-x)\log_2(1-x), \qquad x \in [0,1].$$

2. Refer to chapter 9, problem 2 in the textbook

Problems and Solutions in Quantum Computing and Quantum Information, 2nd edition.

Find $\mathbf{x} \in \mathbb{C}^4$ such that

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} + \mathbf{x}$$

is separable on $\mathbb{C}^2 \otimes \mathbb{C}^2$. Is $\mathbf{x}$ entangled?

3. Let $|\psi\rangle \in \mathbb{C}^n$ be normalized. The fidelity of $|\psi\rangle$ and the density operator ρ is

$$F(\rho, |\psi\rangle) := \sqrt{\langle\psi|\rho|\psi\rangle}.$$

Let X_1, X_2, ..., X_m be $n \times n$ unitary operators and let

$$\rho := p_0|\psi\rangle\langle\psi| + \sum_{j=1}^{m} p_j\, X_j|\psi\rangle\langle\psi|X_j^*, \qquad p_0, p_1, \ldots, p_m \in [0,1],\ \sum_{j=0}^{m} p_m = 1.$$

Show that

$$F(\rho, |\psi\rangle) = \sqrt{p_0 + \sum_{j=1}^{m} p_j |\langle\psi|X_j|\psi\rangle|^2}.$$

Let $\{\, |0\rangle, |1\rangle, \}$ denote an orthonormal basis in $\mathbb{C}^2$.

Calculate the minimum fidelity for $m = 1$, $n = 2$, and $|\psi\rangle = |0\rangle$ and $X_1 = \sigma_x = U_{NOT}$.

Calculate the minimum fidelity for $m = 1$, $n = 2$, $p_0 = p_1 = \frac{1}{2}$,

$$|\psi\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle, \qquad \theta \in [0, 2\pi]$$

and $X_1 = \sigma_x = U_{NOT}$.

Calculate the minimum fidelity for $m = 1$, $n = 8$, $p_0 = p_1 = \frac{1}{2}$,

$$|\psi\rangle = \cos\theta\,|0\rangle \otimes |0\rangle \otimes |0\rangle + \sin\theta\,|1\rangle \otimes |1\rangle \otimes |1\rangle, \qquad \theta \in [0, 2\pi]$$

and $X_1 = I_2 \otimes I_2 \otimes \sigma_x$.