UNIVERSITEIT VAN JOHANNESBURG

Toegepaste Wiskunde 3B

Taak #1 **7:30, 21 Julie 2009**

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1. Vind twee verskillende 2×2 matrikse oor die reële getalle met eiewaardes 1 en -1.

2. Vind 'n 2×2 matriks oor die reële getalle met net een 1-dimensionele eievektorruimte, d.w.s. alle eievektore is lineêr afhanklik. Dit beteken dat, gegee 'n eievektor van die matriks, enige vektor wat ortogonaal aan die eievektor is sal nie 'n eievektor van die matriks wees nie.

3. Vind alle 2×2 matrikse oor die reële getalle met eiewaardes 0 en 1.

Wenk: Die karakteristieke vergelyking vir die eiewaardes λ van die 2×2 matriks A is

$$\lambda^2 - \mathrm{tr}(A)\lambda + \det(A) = 0.$$

4. Wervys na taak 1 (2008). Laat

$$A := \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

Bepaal die eiewaardes λ_1 en λ_2. Bepaal die ooreenstemmende genormaliseerde eievektore $\mathbf{x}_1$ en $\mathbf{x}_2$. Bereken

$$\lambda_1 \mathbf{x}_1 \mathbf{x}_1^* + \lambda_2 \mathbf{x}_2 \mathbf{x}_2^*$$

waar $\mathbf{x}^*$ die getransponeerde komplekse toegevoegde van $\mathbf{x}$ is.

Applied Mathematics 3B

Assignment #1 **7:30, 21 July 2009**

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1. Find two different 2×2 matrices over the real numbers with eigenvalues 1 and -1.

2. Find a 2×2 matrix over the real numbers with only one 1-dimensional eigenspace, i.e. all eigenvectors are linearly dependent. This means that given an eigenvector of the matrix, any vector which is orthogonal (in the 2-dimensional Euclidean space) to the eigenvector is not an eigenvector of the matrix.

3. Find all 2×2 matrices over the real numbers with eigenvalues 0 and 1.

Hint: The characteristic equation for eigenvalues λ of the 2×2 matrix A is

$$\lambda^2 - \mathrm{tr}(A)\lambda + \det(A) = 0.$$

4. Refer to assignment 1 (2008). Let

$$A := \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

Determine the eigenvalues λ_1 and λ_2. Determine the corresponding orthonormal eigenvectors $\mathbf{x}_1$ and $\mathbf{x}_2$. Calculate

$$\lambda_1 \mathbf{x}_1 \mathbf{x}_1^* + \lambda_2 \mathbf{x}_2 \mathbf{x}_2^*$$

where $\mathbf{x}^*$ denotes the complex conjugate and transpose of $\mathbf{x}$.
