# University of Johannesburg

## Applied Mathematics 3B

**Assignment #9** **Solutions**

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1. We have $w = e^{2\pi i/4} = e^{\pi i/2}$ and

$$\begin{aligned} F_4 &= \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{\pi i/2} & e^{2\pi i/2} & e^{3\pi i/2} \\ 1 & e^{2\pi i/2} & e^{4\pi i/2} & e^{6\pi i/2} \\ 1 & e^{3\pi i/2} & e^{6\pi i/2} & e^{9\pi i/2} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix} \end{aligned}$$

   Consequently for the first case we find

$$\begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -2 \\ 0 \end{pmatrix}$$

   and for the second case

$$\begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \\ 0 \end{pmatrix}.$$

   The nonzero $\hat{x}_k$ identify frequencies, thus $n/k$ yields the periodicity. In both cases we find periodicity $4/0 \to 4/4 = 1$ and $4/2 = 2$. The underlying periodicity is 2.

2. We have

$$\begin{aligned} \left(\frac{F_n F_n^*}{n}\right)_{j,k} &= \frac{1}{n}\sum_{l=0}^{n-1} w^{jl}(w^*)^{kl} \\ &= \frac{1}{n}\sum_{l=0}^{n-1} e^{(j-k)2l\pi i/n} \\ &= \begin{cases} 1 & j = k \\ \frac{1-e^{n(j-k)2l\pi i/n}}{1-e^{(j-k)2l\pi i/n}} & j \neq k \end{cases} \\ &= \delta_{jk} \end{aligned}$$

   and

$$\begin{aligned} \left(\frac{F_n^* F_n}{n}\right)_{j,k} &= \frac{1}{n}\sum_{l=0}^{n-1} w^{kl}(w^*)^{jl} \\ &= \frac{1}{n}\sum_{l=0}^{n-1} e^{(k-j)2l\pi i/n} \\ &= \begin{cases} 1 & j = k \\ \frac{1-e^{n(k-j)2l\pi i/n}}{1-e^{(k-j)2l\pi i/n}} & j \neq k \end{cases} \\ &= \delta_{kj}. \end{aligned}$$

   Consequently $F_n F_n^*/n = F_n^* F_n = I$ and $F_n^*/n = F_n^{-1}$.