Universiteit van Johannesburg

Toegepaste Wiskunde 3B

Taak #9 **7:30, 30 September 2008**

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1. Die diskrete Fouriertransformasie oor n punte kan in matriks vorm geskryf word

$$F_n := \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & w & w^2 & \dots & w^{n-1} \\ 1 & w^2 & w^4 & \dots & w^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & w^{n-1} & w^{2(n-1)} & \dots & w^{(n-1)^2} \end{pmatrix}$$

waar $w = e^{2\pi i/n}$ is die n-te wortel van 1. Die diskrete Fourier transformasie is

$$(\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n)^T = F_n(x_1, x_2, \dots, x_n)^T.$$

Bereken $F_4(0,1,0,1)^T$ en $F_4(1,0,1,0)^T$. Interpreteer die resultate om die onderliggende periodisiteit te vind.

2. Die diskrete Fouriertransformasie oor n punte kan in matriks vorm geskryf word

$$F_n := \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & w & w^2 & \dots & w^{n-1} \\ 1 & w^2 & w^4 & \dots & w^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & w^{n-1} & w^{2(n-1)} & \dots & w^{(n-1)^2} \end{pmatrix}$$

waar $w = e^{2\pi i/n}$ is die n-te wortel van 1. Wys dat die inverse F_n^{-1} deur

$$F_n^{-1} := \frac{F_n^*}{n} = \frac{1}{n} \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & (w^*) & (w^*)^2 & \dots & (w^*)^{n-1} \\ 1 & (w^*)^2 & (w^*)^4 & \dots & (w^*)^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & (w^*)^{n-1} & (w^*)^{2(n-1)} & \dots & (w^*)^{(n-1)^2} \end{pmatrix}$$

gegee is, waar $w^* = e^{-2\pi i/n}$.

University of Johannesburg

Applied Mathematics 3B

Assignment #9 **7:30, 30 September 2008**

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1. The discrete Fourier transform over n points can be written in matrix form

$$F_n := \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & w & w^2 & \dots & w^{n-1} \\ 1 & w^2 & w^4 & \dots & w^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & w^{n-1} & w^{2(n-1)} & \dots & w^{(n-1)^2} \end{pmatrix}$$

where $w = e^{2\pi i/n}$ is the n-th root of unity. We obtain the discrete Fourier transform from

$$(\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n)^T = F_n(x_1, x_2, \dots, x_n)^T.$$

Calculate $F_4(0,1,0,1)^T$ and $F_4(1,0,1,0)^T$. Interpret the results to find the underlying periodicity.

2. The discrete Fourier transform over n points can be written in matrix form

$$F_n := \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & w & w^2 & \dots & w^{n-1} \\ 1 & w^2 & w^4 & \dots & w^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & w^{n-1} & w^{2(n-1)} & \dots & w^{(n-1)^2} \end{pmatrix}$$

where $w = e^{2\pi i/n}$ is the n-th root of unity. Show that the inverse F_n^{-1} is given by

$$F_n^{-1} := \frac{F_n^*}{n} = \frac{1}{n} \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & (w^*) & (w^*)^2 & \dots & (w^*)^{n-1} \\ 1 & (w^*)^2 & (w^*)^4 & \dots & (w^*)^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & (w^*)^{n-1} & (w^*)^{2(n-1)} & \dots & (w^*)^{(n-1)^2} \end{pmatrix}$$

where $w^* = e^{-2\pi i/n}$.