

Universiteit van Johannesburg

Toegepaste Wiskunde 3B

Taak #8

7:30, 23 September 2008

1. Laat

$$|\psi\rangle := \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad |\phi\rangle := \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Vind 'n unitêre matriks U sodat $|\psi\rangle = U|\phi\rangle$.

2. Laat $|\psi\rangle, |\phi\rangle \in \mathbb{C}^2$. Vind 'n unitêre matriks U sodat $|\psi\rangle = U|\phi\rangle$.

(a) Skryf U in terme van die komponente van $|\psi\rangle$ en $|\phi\rangle$.

(b) Skryf U in terme van $|\psi\rangle, |\phi\rangle, |\psi_\perp\rangle$ en $|\phi_\perp\rangle$ waar

$$\langle \psi_\perp | \psi_\perp \rangle = \langle \phi_\perp | \phi_\perp \rangle = 1, \quad \langle \psi | \psi_\perp \rangle = \langle \phi | \phi_\perp \rangle = 0.$$

3. Bepaal

$$U_{QFT,n} := \frac{1}{\sqrt{2^n}} \sum_{j=1}^{2^n} \sum_{k=1}^{2^n} e^{-i2\pi kj/2^n} \mathbf{e}_{k,2^n} \mathbf{e}_{j,2^n}^*$$

vir $n = 1$ en $n = 2$. Wat is die verhouding tussen $U_{QFT,1}$ en $U_{QFT,2}$?

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Applied Mathematics 3B

Assignment #8

7:30, 23 September 2008

1. Let

$$|\psi\rangle := \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad |\phi\rangle := \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Find a unitary matrix U such that $|\psi\rangle = U|\phi\rangle$.

2. Let $|\psi\rangle, |\phi\rangle \in \mathbb{C}^2$. Find a unitary matrix U such that $|\psi\rangle = U|\phi\rangle$.

(a) Express U in terms of the components of $|\psi\rangle$ and $|\phi\rangle$.

(b) Express U in terms of $|\psi\rangle$, $|\phi\rangle$, $|\psi_\perp\rangle$ and $|\phi_\perp\rangle$ where

$$\langle\psi_\perp|\psi_\perp\rangle = \langle\phi_\perp|\phi_\perp\rangle = 1, \quad \langle\psi|\psi_\perp\rangle = \langle\phi|\phi_\perp\rangle = 0.$$

3. Determine

$$U_{QFT,n} := \frac{1}{\sqrt{2^n}} \sum_{j=1}^{2^n} \sum_{k=1}^{2^n} e^{-i2\pi kj/2^n} \mathbf{e}_{k,2^n} \mathbf{e}_{j,2^n}^*$$

for $n = 1$ and $n = 2$. What is the relation between $U_{QFT,1}$ and $U_{QFT,2}$?
