# Universiteit van Johannesburg

## Toegepaste Wiskunde 3B

**Taak #7** **7:30, 16 September 2008**

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**1.** Laat

$$|\psi\rangle := \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \qquad |\phi\rangle := \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}.$$

Is $|\psi\rangle$ en $|\phi\rangle$ genormaliseerd en ortogonaal? Laat $p \in [0,1]$ en

$$\rho := p|\psi\rangle\langle\psi| + (1-p)|\phi\rangle\langle\phi|.$$

Bereken $\mathrm{tr}\rho$ en $\mathrm{tr}(|\psi\rangle\langle\psi|\rho)$. Is die eiewaardes van $\rho$ nie-negatief?

**Wenk:** Laat $\lambda$ 'n eiewaarde van $\rho$ wees en laat $|x\rangle$ 'n ooreenstemmende genormaliseerde eievektor wees. Dan is

$$\langle x|\rho|x\rangle = \lambda\langle x|x\rangle = \lambda.$$

**2.** Laat

$$|\psi\rangle := \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \qquad |\phi\rangle := \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}.$$

Laat

$$\rho := |\psi\rangle\langle\psi| \otimes |\phi\rangle\langle\phi|.$$

Bereken $\mathrm{tr}\rho$. Bereken

$$\rho_1 := \sum_{j=1}^{2}(I_2 \otimes \mathbf{e}_{j,2}^T)\rho(I_2 \otimes \mathbf{e}_{j,2}), \qquad \rho_2 := \sum_{j=1}^{2}(\mathbf{e}_{j,2}^T \otimes I_2)\rho(\mathbf{e}_{j,2} \otimes I_2).$$

Bereken $\mathrm{tr}\rho_1$ en $\mathrm{tr}\rho_2$.

**3.** Laat

$$|\psi\rangle := \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \qquad |\phi\rangle := \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}.$$

Laat $p \in [0,1]$ en

$$\rho := p|\psi\rangle\langle\psi| \otimes |\phi\rangle\langle\phi| + (1-p)|\phi\rangle\langle\phi| \otimes |\psi\rangle\langle\psi|.$$

Bereken $\mathrm{tr}\rho$. Bereken

$$\rho_1 := \sum_{j=1}^{2}(I_2 \otimes \mathbf{e}_{j,2}^T)\rho(I_2 \otimes \mathbf{e}_{j,2}), \qquad \rho_2 := \sum_{j=1}^{2}(\mathbf{e}_{j,2}^T \otimes I_2)\rho(\mathbf{e}_{j,2} \otimes I_2).$$

Bereken $\mathrm{tr}\rho_1$ en $\mathrm{tr}\rho_2$.

# University of Johannesburg

## Applied Mathematics 3B

**Assignment #7** **7:30, 16 September 2008**

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**1.** Let

$$|\psi\rangle := \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \qquad |\phi\rangle := \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}.$$

Are $|\psi\rangle$ and $|\phi\rangle$ normalized and orthogonal? Let $p \in [0,1]$ and

$$\rho := p|\psi\rangle\langle\psi| + (1-p)|\phi\rangle\langle\phi|.$$

Calculate $\mathrm{tr}\rho$ and $\mathrm{tr}(|\psi\rangle\langle\psi|\rho)$. Are the eigenvalues of $\rho$ non-negative?

**Hint:** Let $\lambda$ be an eigenvalue of $\rho$ and let $|x\rangle$ be a corresponding normalized eigenvector. Then

$$\langle x|\rho|x\rangle = \lambda\langle x|x\rangle = \lambda.$$

**2.** Let

$$|\psi\rangle := \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \qquad |\phi\rangle := \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}.$$

Let

$$\rho := |\psi\rangle\langle\psi| \otimes |\phi\rangle\langle\phi|.$$

Calculate $\mathrm{tr}\rho$. Calculate

$$\rho_1 := \sum_{j=1}^{2}(I_2 \otimes \mathbf{e}_{j,2}^T)\rho(I_2 \otimes \mathbf{e}_{j,2}), \qquad \rho_2 := \sum_{j=1}^{2}(\mathbf{e}_{j,2}^T \otimes I_2)\rho(\mathbf{e}_{j,2} \otimes I_2).$$

Calculate $\mathrm{tr}\rho_1$ and $\mathrm{tr}\rho_2$.

**3.** Let

$$|\psi\rangle := \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \qquad |\phi\rangle := \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}.$$

Let $p \in [0,1]$ and

$$\rho := p|\psi\rangle\langle\psi| \otimes |\phi\rangle\langle\phi| + (1-p)|\phi\rangle\langle\phi| \otimes |\psi\rangle\langle\psi|.$$

Calculate $\mathrm{tr}\rho$. Calculate

$$\rho_1 := \sum_{j=1}^{2}(I_2 \otimes \mathbf{e}_{j,2}^T)\rho(I_2 \otimes \mathbf{e}_{j,2}), \qquad \rho_2 := \sum_{j=1}^{2}(\mathbf{e}_{j,2}^T \otimes I_2)\rho(\mathbf{e}_{j,2} \otimes I_2).$$

Calculate $\mathrm{tr}\rho_1$ and $\mathrm{tr}\rho_2$.

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