

Universiteit van Johannesburg

Toegepaste Wiskunde 3B

Taak #6

7:30, 9 September 2008

1. Laat

$$A := \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad B := \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \quad \text{en} \quad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

waar $a_{ij}, b_{ij} \in \mathbb{R}$, $j, k \in \{1, 2\}$. Laat

$$\left\{ \mathbf{e}_{1,2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \mathbf{e}_{2,2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}, \quad \left\{ \mathbf{e}_{1,4} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{e}_{2,4} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{e}_{3,4} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{e}_{4,4} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

die standaardbasis in \mathbb{R}^2 en in \mathbb{R}^4 respektiewelik wees. Bereken

$$\text{vec}_{2 \times 2} A := \sum_{j=1}^2 (A \mathbf{e}_{j,2}) \otimes \mathbf{e}_{j,2}, \quad \text{vec}_{2 \times 2} B := \sum_{j=1}^2 (B \mathbf{e}_{j,2}) \otimes \mathbf{e}_{j,2}$$

$$\text{vec}_{4 \times 4} A \otimes B := \sum_{j=1}^4 ((A \otimes B) \mathbf{e}_{j,4}) \otimes \mathbf{e}_{j,4}$$

Bereken

$$\sum_{j=1}^2 \mathbf{e}_{j,2} \otimes I_2 \otimes (A \mathbf{e}_{j,2}) \otimes I_2, \quad \sum_{j=1}^2 I_2 \otimes \mathbf{e}_{j,2} \otimes I_2 \otimes (B \mathbf{e}_{j,2}).$$

University of Johannesburg

Applied Mathematics 3B

Assignment #6

7:30, 9 September 2008

1. Let

$$A := \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad B := \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \quad \text{and} \quad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

where $a_{ij}, b_{ij} \in \mathbb{R}$, $j, k \in \{1, 2\}$. Let

$$\left\{ \mathbf{e}_{1,2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \mathbf{e}_{2,2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}, \quad \left\{ \mathbf{e}_{1,4} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{e}_{2,4} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{e}_{3,4} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{e}_{4,4} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

denote the the standard basis in \mathbb{R}^2 and \mathbb{R}^4 respectively. Calculate

$$\text{vec}_{2 \times 2} A := \sum_{j=1}^2 (A \mathbf{e}_{j,2}) \otimes \mathbf{e}_{j,2}, \quad \text{vec}_{2 \times 2} B := \sum_{j=1}^2 (B \mathbf{e}_{j,2}) \otimes \mathbf{e}_{j,2}$$

$$\text{vec}_{4 \times 4} A \otimes B := \sum_{j=1}^4 ((A \otimes B) \mathbf{e}_{j,4}) \otimes \mathbf{e}_{j,4}$$

Calculate

$$\sum_{j=1}^2 \mathbf{e}_{j,2} \otimes I_2 \otimes (A \mathbf{e}_{j,2}) \otimes I_2, \quad \sum_{j=1}^2 I_2 \otimes \mathbf{e}_{j,2} \otimes I_2 \otimes (B \mathbf{e}_{j,2}).$$
