Universiteit van Johannesburg

Toegepaste Wiskunde 3B

Taak #5 **7:30, 19 Augustus 2008**

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1. Laat $\{\,|0\rangle,\,|1\rangle\,\}$ 'n ortonormale basis in $\mathbb{C}^2$ wees. Met ander woorde

$$\langle 0|1\rangle = \langle 1|0\rangle = 0, \qquad \langle 0|0\rangle = \langle 1|1\rangle = 1.$$

Laat $a, b \in \mathbb{C}$ met

$$|a|^2 + |b|^2 = 1$$

en

$$|0'\rangle := a|0\rangle + b|1\rangle, \qquad |1'\rangle := \overline{a}|1\rangle - \overline{b}|0\rangle.$$

(a) Wys dat $\{\,|0'\rangle,\,|1'\rangle\,\}$ 'n ortonormale basis in $\mathbb{C}^2$ is.

(b) Beskou

$$|\psi\rangle := \frac{1}{\sqrt{2}}(|0'\rangle \otimes |1'\rangle - |1'\rangle \otimes |0'\rangle)$$

Herskryf $|\psi\rangle$ in terme van $|0\rangle$ en $|1\rangle$.

(c) Wys dat, vir $\alpha, \beta \in \mathbb{R}$,

$$A := \alpha|0\rangle\langle 0| + \beta|1\rangle\langle 1|$$

'n waarneembare kwantiteit beskryf. Beskryf die uitkomste en geassosieerde waarskynlikhede vir meting van die *eerste* qubit van die twee qubit stelsel beskryf deur $|\psi\rangle$ beskryf deur A.

(d) Laat $|\phi\rangle$ die toestand van die stelsel na meting van die eerste qubit in (c) wees. Beskryf die uitkomste en geassosieerde waarskynlikhede vir meting van die *tweede* qubit van $|\phi\rangle$ beskryf deur A.

2. Verwys na hoofstuk 3, probleem 6 in die handboek

Problems and Solutions in Quantum Computing and Quantum Information, 2de uitgawe.

Vind die singuliere waarde dekomposisie van

(a) $\begin{pmatrix} 1 & -1 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

3. Laat

$$A := \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

waar $a_{11}, a_{12}, a_{21}, a_{22} \in \mathbb{R}$. Wys dat

$$\left\{ \phi_1 = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} \right\}, \qquad \theta \in \mathbb{R}.$$

'n ortonormale basis vir $\mathbb{R}^2$ is. Bereken

$$\sum_{j=1}^{2} \phi_j^T \, A \, \phi_j.$$

Wat is u gevolgtrekking?

University of Johannesburg

Applied Mathematics 3B

Assignment #5 **7:30, 19 August 2008**

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1. Let $\{\, |0\rangle,\ |1\rangle \,\}$ denote an orthonormal basis in $\mathbb{C}^2$. In other words

$$\langle 0|1\rangle = \langle 1|0\rangle = 0, \qquad \langle 0|0\rangle = \langle 1|1\rangle = 1.$$

Let $a, b \in \mathbb{C}$ with

$$|a|^2 + |b|^2 = 1$$

and

$$|0'\rangle := a|0\rangle + b|1\rangle, \qquad |1'\rangle := \overline{a}|1\rangle - \overline{b}|0\rangle.$$

(a) Show that $\{\, |0'\rangle,\ |1'\rangle \,\}$ is an orthonormal basis in $\mathbb{C}^2$.

(b) Consider

$$|\psi\rangle := \frac{1}{\sqrt{2}}(|0'\rangle \otimes |1'\rangle - |1'\rangle \otimes |0'\rangle)$$

Express $|\psi\rangle$ in terms of $|0\rangle$ and $|1\rangle$.

(c) Show that, for $\alpha, \beta \in \mathbb{R}$,

$$A := \alpha|0\rangle\langle 0| + \beta|1\rangle\langle 1|$$

is an observable. Describe the measurement outcomes and associated probabilities when measuring the *first* qubit of the two qubit system described by $|\psi\rangle$ described by A.

(d) Let $|\phi\rangle$ be the state of the system after the measurement in (c). Describe the measurement outcomes and associated probabilities when measuring the *second* qubit of $|\phi\rangle$ described by A.

2. Refer to chapter 3, problem 6 in the textbook

Problems and Solutions in Quantum Computing and Quantum Information, 2nd edition.

Find the singular value decomposition of

(a) $\begin{pmatrix} 1 & -1 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

3. Let

$$A := \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

where $a_{11}, a_{12}, a_{21}, a_{22} \in \mathbb{R}$. Show that

$$\left\{ \phi_1 = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} \right\}, \qquad \theta \in \mathbb{R}.$$

is an orthonormal basis for $\mathbb{R}^2$. Calculate

$$\sum_{j=1}^{2} \phi_j^T\, A\, \phi_j.$$

What can you conclude?
