Universiteit van Johannesburg

Toegepaste Wiskunde 3B

Taak #4 **7:30, 12 Augustus 2008**

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1. Laat

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \qquad \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

waar $x_1, x_2, x_3, y_1, y_2, y_3 \in \mathbb{R}$. Bereken

$$\mathbf{x} \otimes \mathbf{y}, \qquad \mathbf{x}^T \otimes \mathbf{y}, \qquad \mathbf{x}^T\mathbf{y}, \qquad \mathbf{x} \otimes \mathbf{y}^T, \qquad \mathbf{x}\mathbf{y}^T.$$

2.

(a) Vind $\mathbf{a} \in \mathbb{R}^2$ en $\mathbf{b} \in \mathbb{R}^2$ wat 'n minimum vir

$$\left\| \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} - \mathbf{a} \otimes \mathbf{b} \right\|^2, \qquad \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \quad a_1, a_2, b_1, b_2 \in \mathbb{R}.$$

gee, waar $\|\mathbf{x}\| := \sqrt{\mathbf{x}^T\mathbf{x}}$ die Euklidiese norm vir $\mathbf{x} \in \mathbb{R}^4$ is.

(b) Vind $\mathbf{a} \in \mathbb{R}^2$ en $\mathbf{b} \in \mathbb{R}^2$ wat 'n minimum vir

$$\left\| \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} - \mathbf{a} \otimes \mathbf{b} \right\|^2, \qquad \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \quad a_1, a_2, b_1, b_2 \in \mathbb{R}.$$

gee, waar $\|\mathbf{x}\| := \sqrt{\mathbf{x}^T\mathbf{x}}$ die Euklidiese norm vir $\mathbf{x} \in \mathbb{R}^4$ is.

3. Laat

$$A := \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \quad \text{en} \quad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

waar $a_{ij} \in \mathbb{R}$, $j, k \in \{1, 2, 3, 4\}$. Laat

$$\left\{ \mathbf{e}_{1,2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \mathbf{e}_{2,2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}, \qquad \left\{ \mathbf{e}_{1,4} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{e}_{2,4} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{e}_{3,4} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{e}_{4,4} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

die standaardbasis in $\mathbb{R}^2$ en in $\mathbb{R}^4$ respektiewelik wees. Bereken

$$\sum_{j=1}^{4} \mathbf{e}_{j,4}^T \, A \, \mathbf{e}_{j,4} \quad \text{en} \quad \sum_{j=1}^{2} (\mathbf{e}_{j,2} \otimes I_2)^T \, A \, (\mathbf{e}_{j,2} \otimes I_2).$$

Wat is u gevolgtrekking?

University of Johannesburg

Applied Mathematics 3B

Assignment #4 **7:30, 12 August 2008**

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1. Let

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \qquad \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

where $x_1, x_2, x_3, y_1, y_2, y_3 \in \mathbb{R}$. Calculate

$$\mathbf{x} \otimes \mathbf{y}, \qquad \mathbf{x}^T \otimes \mathbf{y}, \qquad \mathbf{x}^T\mathbf{y}, \qquad \mathbf{x} \otimes \mathbf{y}^T, \qquad \mathbf{x}\mathbf{y}^T.$$

2.

(a) Find $\mathbf{a} \in \mathbb{R}^2$ and $\mathbf{b} \in \mathbb{R}^2$ which minimizes

$$\left\| \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} - \mathbf{a} \otimes \mathbf{b} \right\|^2, \qquad \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \quad a_1, a_2, b_1, b_2 \in \mathbb{R}.$$

where $\|\mathbf{x}\| := \sqrt{\mathbf{x}^T\mathbf{x}}$ denotes the Euclidean norm for $\mathbf{x} \in \mathbb{R}^4$.

(b) Find $\mathbf{a} \in \mathbb{R}^2$ and $\mathbf{b} \in \mathbb{R}^2$ which minimizes

$$\left\| \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} - \mathbf{a} \otimes \mathbf{b} \right\|^2, \qquad \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \quad a_1, a_2, b_1, b_2 \in \mathbb{R}.$$

where $\|\mathbf{x}\| := \sqrt{\mathbf{x}^T\mathbf{x}}$ denotes the Euclidean norm for $\mathbf{x} \in \mathbb{R}^4$.

3. Let

$$A := \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \quad \text{and} \quad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

where $a_{ij} \in \mathbb{R}$, $j, k \in \{1, 2, 3, 4\}$. Let

$$\left\{ \mathbf{e}_{1,2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \mathbf{e}_{2,2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}, \qquad \left\{ \mathbf{e}_{1,4} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{e}_{2,4} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{e}_{3,4} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{e}_{4,4} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

denote the the standard basis in $\mathbb{R}^2$ and $\mathbb{R}^4$ respectively. Calculate

$$\sum_{j=1}^{4} \mathbf{e}_{j,4}^T \, A \, \mathbf{e}_{j,4} \quad \text{and} \quad \sum_{j=1}^{2} (\mathbf{e}_{j,2} \otimes I_2)^T \, A \, (\mathbf{e}_{j,2} \otimes I_2).$$

What can you conclude?
