# Universiteit van Johannesburg

## Toegepaste Wiskunde 3B

**Taak #3** **7:30, 5 Augustus 2008**

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**1.** Beskou die Heisenberg vergelyking van beweging

$$-i\hbar\frac{dA(t)}{dt} = [\hat{H}, A(t)]$$

waar $\hat{H}$ onafhanklik van $t$ is, en

$$[M, N] := MN - NM$$

is die kommutator. Gegee $A(0)$, vind die Tayloruitbreiding vir $A(t)$ om $t = 0$.

**Wenk.** Dit is nuttig om te definieer

$$[M, N]_0 := N, \qquad [M, N]_{n+1} := [M, [M, N]_n].$$

Neem kennis dat

$$-i\hbar\frac{d^2A(t)}{dt^2} = \left[\hat{H}, \frac{dA(t)}{dt}\right].$$

**2.** Verwys na vraag 1. Laat $A(0) = \sigma_x$. Gebruik die Tayloruitbreiding om $A(t)$ te vind met

$$(a) \quad \hat{H} = I_2, \qquad (b) \quad \hat{H} = \sigma_x, \qquad (c) \quad \hat{H} = \sigma_z$$

respektiewelik.

**3.** Laat $\hat{q}$ die posisie operator wees en $\hat{p}$ die momentum operator wees gedefineer as volg

$$\hat{q}f := xf$$

$$\hat{p}f := -i\hbar\frac{df}{dx}$$

waar $x, f : \mathbb{R} \to \mathbb{R}$. Omdat die operasies nie assosiatief is nie gebruik ons regs assosiatiwitet in ons berkenings, d.w.s.

$$ABC := A(BC).$$

Vind die eiewaardes van $\hat{q}$ and $\hat{p}$. Bereken

$$[\hat{p}, \hat{q}]f := \hat{p}\hat{q}f - \hat{q}\hat{p}f$$

waar $f : \mathbb{R} \to \mathbb{R}$.

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# University of Johannesburg

## Applied Mathematics 3B

**Assignment #3** **7:30, 5 August 2008**

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**1.** Consider the Heisenberg equation of motion

$$-i\hbar\frac{dA(t)}{dt} = [\hat{H}, A(t)]$$

where $\hat{H}$ does not depend on $t$, and

$$[M, N] := MN - NM$$

is the commutator. Given $A(0)$, find the Taylor series for $A(t)$ around $t = 0$.

**Hint.** It is useful to define

$$[M, N]_0 := N, \qquad [M, N]_{n+1} := [M, [M, N]_n].$$

Note that

$$-i\hbar\frac{d^2A(t)}{dt^2} = \left[\hat{H}, \frac{dA(t)}{dt}\right].$$

**2.** Refer to question 1. Let $A(0) = \sigma_x$. Use the Taylor expansion to find $A(t)$ with

$$(a) \quad \hat{H} = I_2, \qquad (b) \quad \hat{H} = \sigma_x, \qquad (c) \quad \hat{H} = \sigma_z$$

respectively.

**3.** Let $\hat{q}$ be the position operator and $\hat{p}$ be the momentum operator defined by

$$\hat{q}f := xf$$

$$\hat{p}f := -i\hbar\frac{df}{dx}$$

where $x, f : \mathbb{R} \to \mathbb{R}$. Since these operations are not associative we use right associativity in our calculations, i.e.

$$ABC := A(BC).$$

Find the eigenvalues of $\hat{q}$ and $\hat{p}$. Calculate

$$[\hat{p}, \hat{q}]f := \hat{p}\hat{q}f - \hat{q}\hat{p}f$$

where $f : \mathbb{R} \to \mathbb{R}$.

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