Universiteit van Johannesburg

Toegepaste Wiskunde 3B

Taak #2 **7:30, 29 Julie 2008**

---

1. Laat $\mathbf{x} := \begin{pmatrix} a & b \end{pmatrix}^*$ 'n genormaliseerde vektor in die Hilbertruimte $\mathbb{C}^2$ wees.

$$|a|^2 + |b|^2 = 1, \qquad a, b \in \mathbb{C}.$$

Wys dat die matriks $\mathbf{x}\mathbf{x}^*$ is 'n digtheids matriks, d.w.s dat die matriks 'n spoor van 1 het en dat al die eiewaardes nie negatief is nie.

2. Wys dat die 2×2 identiteits matriks I_2 saam met die Pauli matrikse

$$\left\{ \frac{I_2}{\sqrt{2}}, \frac{\sigma_x}{\sqrt{2}}, \frac{\sigma_y}{\sqrt{2}}, \frac{\sigma_z}{\sqrt{2}} \right\}$$

'n basis vir die vektorruimte van 2×2 matrikse oor $\mathbb{C}$ vorm. Verder, wys dat die versameling 'n ortonormale basis is met betrekking tot die Hilbert-Schmidt skalaarproduk

$$\langle A, B \rangle := \mathrm{tr}(AB^*).$$

Brei die matriks

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}, \qquad a, b, c, d \in \mathbb{C}$$

uit in terme van die basis.

3. Bereken

$$e^A := \sum_{j=0}^{\infty} \frac{A^j}{j!}$$

waar

$$A := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

University of Johannesburg

Applied Mathematics 3B

Assignment #2 **7:30, 29 July 2008**

---

1. Let $\mathbf{x} := (\,a \quad b\,)^*$ be a normalized vector in the Hilbert space $\mathbb{C}^2$.

$$|a|^2 + |b|^2 = 1, \qquad a, b \in \mathbb{C}.$$

Show that the matrix $\mathbf{x}\mathbf{x}^*$ is a density matrix, i.e. that the matrix has unit trace and that all eigenvalues are non-negative.

2. Show that the 2×2 identity matrix I_2 together with the Pauli matrices

$$\left\{ \frac{I_2}{\sqrt{2}}, \frac{\sigma_x}{\sqrt{2}}, \frac{\sigma_y}{\sqrt{2}}, \frac{\sigma_z}{\sqrt{2}} \right\}$$

forms a basis for the vector space of 2×2 matrices over $\mathbb{C}$. Furthermore, show that the set is an orthonormal basis with respect to the Hilbert-Schmidt scalar product

$$\langle A, B \rangle := \mathrm{tr}(AB^*).$$

Expand the matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}, \qquad a, b, c, d \in \mathbb{C}$$

in terms of this basis.

3. Calculate

$$e^A := \sum_{j=0}^{\infty} \frac{A^j}{j!}$$

where

$$A := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$
