Universiteit van Johannesburg

Toegepaste Wiskunde 3B

Taak #10 **7:30, 14 Oktober 2008**

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1. Laat

$$\{\, |0\rangle,\ |1\rangle,\ |2\rangle,\ |3\rangle \,\}$$

'n ortonormale basis in $\mathbb{C}^4$ wees. Pas die kwantum Fourier-transformasie toe op die toestand

$$|\psi_1\rangle := \frac{1}{2}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle + |2\rangle \otimes |0\rangle + |3\rangle \otimes |1\rangle),$$

en op die toestand

$$|\psi_2\rangle := \frac{1}{2}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle + |2\rangle \otimes |1\rangle + |3\rangle \otimes |0\rangle)$$

d.w.s. pas $U_{QFT,4} \otimes I_4$ toe. Die kwantum Fourier-transformasie in $\mathbb{C}^4$ is

$$U_{QFT,4} = \frac{1}{2}\sum_{j=0}^{3}\sum_{k=0}^{3} e^{-i2\pi jk/4}|j\rangle\langle k|.$$

Gebruik u antwoorde om die periodisiteit van die ry van toestande in $|\psi_1\rangle$ (0101) en $|\psi_2\rangle$ (1010) te analiseer.

2. Die inverse om die gemiddeld operasie op n qubits is deur

$$U_{IA,n} = \underbrace{(U_H \otimes U_H \otimes \cdots \otimes U_H)}_{n \text{ times}} \operatorname{diag}(1, \underbrace{-1, -1, \ldots, -1}_{2^n-1 \text{ times}}) \underbrace{(U_H \otimes U_H \otimes \cdots \otimes U_H)}_{n \text{ times}}.$$

Bereken $U_{IA,1}$ en $U_{IA,2}$. Bereken

$$U_{IA,1}\begin{pmatrix} a \\ b \end{pmatrix} \quad \text{en} \quad U_{IA,2}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

where $a, b, c, d \in \mathbb{R}$.

3. Laat $\{\, |0\rangle,\ |1\rangle \,\}$ en $\{\, |a\rangle,\ |b\rangle \,\}$ twee ortonormale basisse in $\mathbb{C}^2$ wees. Gebruik die uitbreidings

$$|0\rangle = \langle a|0\rangle|a\rangle + \langle b|0\rangle|b\rangle, \qquad |1\rangle = \langle a|1\rangle|a\rangle + \langle b|1\rangle|b\rangle$$

om die ortonormaliteit van $\{\, |0\rangle,\ |1\rangle \,\}$ en die toestand

$$|\psi\rangle := \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle)$$

in terme van $|a\rangle$ en $|b\rangle$ uit te druk.

Ons meet die toestand $|\psi\rangle$ met betrekking tot die waarneembaar $I_2 \otimes |1\rangle\langle 1|$ of met betrekking tot die waarneembaar $I_2 \otimes |b\rangle\langle b|$ met dieselfde waarskynlikheid (d.w.s. $\frac{1}{2}$). Die uitkomstes in beide geval is 0 en 1. Daarna meet ons met betrekking tot die waarneembaar $|1\rangle\langle 1| \otimes I_2$ of met betrekking tot die waarneembaar $|b\rangle\langle b| \otimes I_2$ met dieselfde waarskynlikheid (d.w.s. $\frac{1}{2}$). Die uitkomstes in beide geval is 0 en 1. Wat is die waarskynlikheid dat die uitkomstes van die twee metings verskillend is?

University of Johannesburg

Applied Mathematics 3B

Assignment #10 **7:30, 14 October 2008**

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1. Let

$$\{\, |0\rangle,\, |1\rangle,\, |2\rangle,\, |3\rangle \,\}$$

denote an orthonormal basis in $\mathbb{C}^4$. Apply the quantum Fourier transform to the state

$$|\psi_1\rangle := \frac{1}{2}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle + |2\rangle \otimes |0\rangle + |3\rangle \otimes |1\rangle),$$

and to the state

$$|\psi_2\rangle := \frac{1}{2}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle + |2\rangle \otimes |1\rangle + |3\rangle \otimes |0\rangle)$$

i.e. apply $U_{QFT,4} \otimes I_4$. The quantum Fourier transform on $\mathbb{C}^4$ is given by

$$U_{QFT,4} = \frac{1}{2}\sum_{j=0}^{3}\sum_{k=0}^{3} e^{-i2\pi jk/4}|j\rangle\langle k|.$$

Use your answers to analyze the periodicity of the sequence of states in $|\psi_1\rangle$ (0101) and $|\psi_2\rangle$ (1010).

2. The inversion about average operation on n qubits is given by

$$U_{IA,n} = (\underbrace{U_H \otimes U_H \otimes \cdots \otimes U_H}_{n \text{ times}})\,\mathrm{diag}(1, \underbrace{-1, -1, \ldots, -1}_{2^n-1 \text{ times}})(\underbrace{U_H \otimes U_H \otimes \cdots \otimes U_H}_{n \text{ times}}).$$

Calculate $U_{IA,1}$ and $U_{IA,2}$. Calculate

$$U_{IA,1}\begin{pmatrix} a \\ b \end{pmatrix} \quad \text{and} \quad U_{IA,2}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

where $a, b, c, d \in \mathbb{R}$.

3. Let $\{\, |0\rangle,\, |1\rangle \,\}$ and $\{\, |a\rangle,\, |b\rangle \,\}$ denote two orthonormal bases in $\mathbb{C}^2$. Use the expansions

$$|0\rangle = \langle a|0\rangle|a\rangle + \langle b|0\rangle|b\rangle, \qquad |1\rangle = \langle a|1\rangle|a\rangle + \langle b|1\rangle|b\rangle$$

to express the orthonormality of $\{\, |0\rangle,\, |1\rangle \,\}$ and to express

$$|\psi\rangle := \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle)$$

in terms of $|a\rangle$ and $|b\rangle$.

We measure the state $|\psi\rangle$ with respect to the observable $I_2 \otimes |1\rangle\langle 1|$ or with respect to the observable $I_2 \otimes |b\rangle\langle b|$ with equal probability (i.e. $\frac{1}{2}$). The measurement outcomes in either case are 0 and 1. We perform a subsequent measurement with respect to the observable $|1\rangle\langle 1| \otimes I_2$ or with respect to the observable $|b\rangle\langle b| \otimes I_2$ with equal probability (i.e. $\frac{1}{2}$). The measurement outcomes in either case are 0 and 1. What is the probability that the measurement outcomes of the two measurements are different?
