TOEGEPASTE WISKUNDE 3B

Semestertoets: 7 Oktober 2008

Tydsduur: 80 minute **Punte:** 30

Instruksies: Beantwoord al die vrae
Alle berekenings moet getoon word
Sakrekenaars mag gebruik word
Alle hoeke word in radiale gemeet
Die voorgeskrewe handboek word toegelaat

Vraag 1

(a) Is die matriks

$$\rho := \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -i & 0 & 0 & 1 \end{pmatrix}$$

'n digtheidsmatriks? (5)

(b) Is die matriks

$$\rho_1 := \sum_{j=1}^{2} (I_2 \otimes \mathbf{e}_j)^T \rho (I_2 \otimes \mathbf{e}_j)$$

waar

$$I_2 := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{e}_1 := \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \mathbf{e}_2 := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

'n digtheidsmatriks? (5)

(c) Laat ρ_1 en ρ_2 twee $n \times n$ digtheidsmatrikse wees.
Is $\rho_1 + \rho_2$ 'n $n \times n$ digtheidsmatriks? Bewys of bewys die teendeel. (1)

(d) Laat ρ_1 en ρ_2 twee $n \times n$ digtheidsmatrikse wees.
Is $\rho_1 \rho_2$ 'n $n \times n$ digtheidsmatriks? Bewys of bewys die teendeel. (4)

(15)

Vraag 2

Laat

$$\{\, |0\rangle, \, |1\rangle, \, |2\rangle \,\}$$

'n ortonormale basis in $\mathbb{C}^3$ wees. Pas die kwantum Fourier-transformasie toe op die toestand

$$|\psi_1\rangle := \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle),$$

en op die toestand

$$|\psi_2\rangle := \frac{1}{\sqrt{3}}(|0\rangle - |1\rangle + |2\rangle).$$

Die kwantum Fourier-transformasie in $\mathbb{C}^3$ is

$$U_{QFT,3} = \frac{1}{\sqrt{3}} \sum_{j=0}^{2} \sum_{k=0}^{2} e^{-i2\pi jk/3} |j\rangle\langle k|.$$

Vraag 2 vervolg

Gebruik u antwoorde om die periodisiteit van die reeks van koëffisiente in $|\psi_1\rangle$ en $|\psi_2\rangle$ te analiseer. **(10)**

Vraag 3

Beskou die reeks van m waardes $s_j \in \{0, 1, 2, \ldots, n-1\}$

$$\mathbf{s} = s_0, s_1, s_2, \ldots, s_{m-1}.$$

Laat

$$\{\, |0\rangle_n,\ |1\rangle_n,\ \ldots,\ |n-1\rangle_n \,\}$$

'n ortonormale basis in $\mathbb{C}^n$ wees en laat

$$\{\, |0\rangle_m,\ |1\rangle_m,\ \ldots,\ |m-1\rangle_m \,\}$$

'n ortonormale basis in $\mathbb{C}^m$ wees. Die reeks $\mathbf{s}$ kan deur die toestand

$$|\mathbf{s}\rangle := \sum_{j=0}^{m-1} |j\rangle_m \otimes |s_j\rangle_n.$$

voorgestel word. Wys dat

$$|\mathbf{s}\rangle = \sum_{k=0}^{n-1} \left(\sum_{j=0}^{m-1} \delta_{s_j,k} |j\rangle_m \right) \otimes |k\rangle_n.$$

Beskryf die verband tussen die periodisiteit van die reekse verkry uit dié toestand

$$\sum_{j=0}^{m-1} \delta_{s_j,k} |j\rangle_m \quad \to \quad \mathbf{s}_k = \delta_{s_0,k}, \delta_{s_1,k}, \ldots, \delta_{s_{m-1},k}$$

en die periodisiteit van $\mathbf{s}$. **(5)**

EINDE VAN VRAESTEL

APPLIED MATHEMATICS 3B

Semester Test: 7 October 2008

Duration: 80 minutes **Marks:** 30

Instructions: Answer all the questions
All calculations must be shown
Pocket calculators are permitted
All angles are measured in radians
The prescribed text book is allowed

Question 1

(a) Is the matrix

$$\rho := \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -i & 0 & 0 & 1 \end{pmatrix}$$

a density matrix? (5)

(b) Is the matrix

$$\rho_1 := \sum_{j=1}^{2} (I_2 \otimes \mathbf{e}_j)^T \rho (I_2 \otimes \mathbf{e}_j)$$

where

$$I_2 := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{e}_1 := \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \mathbf{e}_2 := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

a density matrix? (5)

(c) Let ρ_1 and ρ_2 be two $n \times n$ density matrices.
Is $\rho_1 + \rho_2$ an $n \times n$ density matrix? Prove or disprove. (1)

(d) Let ρ_1 and ρ_2 be two $n \times n$ density matrices.
Is $\rho_1 \rho_2$ an $n \times n$ density matrix? Prove or disprove. (4)

(15)

Question 2

Let

$$\{\, |0\rangle,\, |1\rangle,\, |2\rangle \,\}$$

denote an orthonormal basis in $\mathbb{C}^3$. Apply the quantum Fourier transform to the state

$$|\psi_1\rangle := \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle),$$

and to the state

$$|\psi_2\rangle := \frac{1}{\sqrt{3}}(|0\rangle - |1\rangle + |2\rangle).$$

The quantum Fourier transform on $\mathbb{C}^3$ is given by

$$U_{QFT,3} = \frac{1}{\sqrt{3}} \sum_{j=0}^{2} \sum_{k=0}^{2} e^{-i2\pi jk/3} |j\rangle\langle k|.$$

Question 2 continues

Use your answers to analyze the periodicity of the sequence of coefficients in $|\psi_1\rangle$ and $|\psi_2\rangle$. **(10)**

Question 3

Consider the sequence of m values $s_j \in \{0, 1, 2, \ldots, n-1\}$

$$\mathbf{s} = s_0, s_1, s_2, \ldots, s_{m-1}.$$

Let

$$\{\, |0\rangle_n,\ |1\rangle_n,\ \ldots,\ |n-1\rangle_n \,\}$$

denote an orthonormal basis in $\mathbb{C}^n$ and let

$$\{\, |0\rangle_m,\ |1\rangle_m,\ \ldots,\ |m-1\rangle_m \,\}$$

denote an orthonormal basis in $\mathbb{C}^m$. The sequence $\mathbf{s}$ can be represented by the state

$$|\mathbf{s}\rangle := \sum_{j=0}^{m-1} |j\rangle_m \otimes |s_j\rangle_n.$$

Show that

$$|\mathbf{s}\rangle = \sum_{k=0}^{n-1} \left(\sum_{j=0}^{m-1} \delta_{s_j,k} |j\rangle_m \right) \otimes |k\rangle_n.$$

Describe the relation between the periodicity of the sequences obtained from this state

$$\sum_{j=0}^{m-1} \delta_{s_j,k} |j\rangle_m \quad \to \quad \mathbf{s}_k = \delta_{s_0,k}, \delta_{s_1,k}, \ldots, \delta_{s_{m-1},k}$$

and the periodicity of $\mathbf{s}$. **(5)**

END OF QUESTION PAPER