UNIVERSITEIT VAN JOHANNESBURG

TOEGEPASTE WISKUNDE 3B

Semestertoets: 26 Augustus 2008

Tydsduur: 80 minute **Punte:** 40

Instruksies: Beantwoord al die vrae
Alle berekenings moet getoon word
Sakrekenaars mag gebruik word
Alle hoeke word in radiale gemeet
Die voorgeskrewe handboek word toegelaat

Vraag 1

Los die differensiaalvergelyking

$$\frac{d\psi}{dt} = \frac{i}{4-2\sqrt{2}} \begin{pmatrix} 3-2\sqrt{2} & 1-\sqrt{2} \\ 1-\sqrt{2} & 1 \end{pmatrix} \psi$$

op vir $\psi(t) \in \mathbb{C}^2$. Vind $\psi(t=\pi)$. Beskou die gevalle

$$\psi(t=0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \psi(t=0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Wenk: Wys dat

$$\left[\frac{1}{4-2\sqrt{2}} \begin{pmatrix} 3-2\sqrt{2} & 1-\sqrt{2} \\ 1-\sqrt{2} & 1 \end{pmatrix} \right]^2 = \frac{1}{4-2\sqrt{2}} \begin{pmatrix} 3-2\sqrt{2} & 1-\sqrt{2} \\ 1-\sqrt{2} & 1 \end{pmatrix}.$$

(10)

Vraag 2

Wys dat die matriks

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & -1 \end{pmatrix}$$

'n waarneembare kwantiteit beskryf. Beskryf die uitkomstes en geassosieerde waarskynlikhede vir die meting van 'n stelsel beskryf deur die toestand

$$\frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Wat is die toestand na die meting? **(10)**

Vraag 3

Vind die singuliere waarde dekomposisie van

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

Is die singuliere waarde dekomposisie uniek? Bewys of bewys die teendeel. **(10)**

Vraag 4 volg

Vraag 4

Kan twee 2×2 matrikse A en B oor $\mathbb{C}$ gevind word sodat

$$A \otimes B = \frac{1}{2}\left(I_2 \otimes I_2 + \sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z\right)?$$

Laat C en D arbitrêre 2×2 matrikse oor $\mathbb{C}$ wees. Los op vir $a, b, c, d \in \mathbb{C}$ sodat

$$C \otimes D = a\, I_2 \otimes I_2 + b\, \sigma_x \otimes \sigma_x + c\, \sigma_y \otimes \sigma_y + d\, \sigma_z \otimes \sigma_z$$

of bewys dat so 'n oplossing nie bestaan nie. **(10)**

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UNIVERSITY OF JOHANNESBURG

APPLIED MATHEMATICS 3B

Semester Test: 26 August 2008

Duration: 80 minutes **Marks:** 40

Instructions: Answer all the questions
All calculations must be shown
Pocket calculators are permitted
All angles are measured in radians
The prescribed text book is allowed

Question 1

Solve the differential equation

$$\frac{d\psi}{dt} = \frac{i}{4-2\sqrt{2}} \begin{pmatrix} 3-2\sqrt{2} & 1-\sqrt{2} \\ 1-\sqrt{2} & 1 \end{pmatrix} \psi$$

for $\psi(t) \in \mathbb{C}^2$. Find $\psi(t=\pi)$. Consider the cases

$$\psi(t=0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \psi(t=0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Hint: Show that

$$\left[\frac{1}{4-2\sqrt{2}} \begin{pmatrix} 3-2\sqrt{2} & 1-\sqrt{2} \\ 1-\sqrt{2} & 1 \end{pmatrix} \right]^2 = \frac{1}{4-2\sqrt{2}} \begin{pmatrix} 3-2\sqrt{2} & 1-\sqrt{2} \\ 1-\sqrt{2} & 1 \end{pmatrix}.$$

(10)

Question 2

Show that the matrix

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & -1 \end{pmatrix}$$

describes an observable. Describe the measurement outcomes and associated probabilities when observing (performing the measurement on) a system described by the state

$$\frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

What is the state after the measurement? **(10)**

Question 3

Find the singular value decomposition of

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

Is the singular value decomposition unique? Prove or disprove. **(10)**

Question 4 follows

Question 4

Can two 2×2 matrices A and B over $\mathbb{C}$ be found such that

$$A \otimes B = \frac{1}{2}\left(I_2 \otimes I_2 + \sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z\right)?$$

Let C and D be arbitrary 2×2 matrices over $\mathbb{C}$. Solve for $a, b, c, d \in \mathbb{C}$ such that

$$C \otimes D = a\, I_2 \otimes I_2 + b\, \sigma_x \otimes \sigma_x + c\, \sigma_y \otimes \sigma_y + d\, \sigma_z \otimes \sigma_z$$

or prove that no such solution exists. **(10)**

END OF QUESTION PAPER