

### TEST IX (TOETS IX) 3B

We use (Ons maak gebruik van)  $\{|0\rangle, |1\rangle\}$  as an orthonormal basis for a 2-dimensional Hilbert space in the following questions (vir 'n ortonormale basis vir 'n 2-dimensionele Hilbertruimte in die volgende vrae).

1) Determine the eigenvalues and eigenvectors of (Bepaal die eiewaardes en eievektore van)

$$A(x) := (1 - x)I + xU_{NOT}, \quad x \in \{0, 1\}.$$

Show that the unitary transform (Toon aan dat die transformasie)

$$U_f = |0f(0)\rangle\langle 00| + |0\overline{f(0)}\rangle\langle 01| + |1f(1)\rangle\langle 10| + |1\overline{f(1)}\rangle\langle 11|$$

where (waar)  $f : \{0, 1\} \rightarrow \{0, 1\}$  is a Boolean function ('n Boole funksie is), can be written as (kan soos volg geskryf word)

$$U_f = |0\rangle\langle 0| \otimes A(f(0)) + |1\rangle\langle 1| \otimes A(f(1)).$$

Calculate (Bereken)

$$U_f \left( I \otimes \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \right).$$

Consider the cases  $f(0) = f(1)$  and  $f(0) \neq f(1)$  (Beskou die gevalle  $f(0) = f(1)$  en  $f(0) \neq f(1)$ ).

2 a) Let  $U$  be a unitary operator which acts on two qubits. (Laat  $U$  'n transformasie, wat op twee qubits werk, wees.) Let  $|\psi\rangle$  and  $|\phi\rangle$  be the states of two arbitrary qubits. (Laat  $|\psi\rangle$  en  $|\phi\rangle$  twee algemene qubits voorstel.) Determine the implications of measuring the first two qubits of (Bepaal die gevolge van meting van die eerste twee qubits van)

$$|\psi\rangle \otimes \frac{1}{\sqrt{2}} (I \otimes U) (|00\rangle + |11\rangle) \otimes |\phi\rangle$$

with respect to the the Bell basis (met betrekking to die Bell basis). How can we obtain  $U|\psi\rangle \otimes |\phi\rangle$  as the last two qubits ? (Beskryf 'n metode om die laaste twee qubits in die staat  $U|\psi\rangle \otimes |\phi\rangle$  te vind.)

2 b) Alice has  $|\psi\rangle$  and Bob has  $|\phi\rangle$ . (Alice het  $|\psi\rangle$  en Bob het  $|\phi\rangle$ .) Describe how  $U$  can be applied to  $|\psi\rangle \otimes |\phi\rangle$  using only classical communication and prior shared entanglement. (Beskryf hoe  $U$  aan  $|\psi\rangle \otimes |\phi\rangle$  toegepas kan word, deur net van klassieke kommunikasie en verstrikking wat vooraf beskikbaar is te gebruik.) After the computation, Alice must still have the first qubit of  $U|\psi\rangle \otimes |\phi\rangle$  and Bob must still have the second qubit of  $U|\psi\rangle \otimes |\phi\rangle$ . (Na die berekening, Alice moet die eerste qubit van  $U|\psi\rangle \otimes |\phi\rangle$  hê en Bob moet die tweede qubit van  $U|\psi\rangle \otimes |\phi\rangle$  hê.)