TEST VIII (TOETS VIII) 3B

We use (Ons maak gebruik van) $\{|0\rangle, |1\rangle\}$ as an orthonormal basis for a 2-dimensional Hilbert space in the following questions (vir 'n ortonormale basis vir 'n 2-dimensionele Hilbertruimte in die volgende vrae).

1) Find all (Vind alle) $x, y, z \in \{0, 1, 2, 3\}$ such that (sodat) $x + y + z = 0 \ mod \ 2$. What are the possible values of (Wat is die moontlike waardes van)

$$f(x, y, z) := \frac{(x + y + z) \ mod \ 4}{2}.$$

when the condition holds (as die eerste vergelyking bevredig is). Now use the binary representation for (Gebruik nou die binêre voorstelling van) $x = x_1x_0$, $y = y_1y_0$ and (en) $z = z_1z_0$ where (waar) $x_0, x_1, y_0, y_1, z_0, z_1 \in \{0, 1\}$. Describe the condition (Beskryf die vergelyking) $x+y+z = 0 \ mod \ 2$ in terms of (in terme van) x_0, x_1, y_0, y_1, z_0 and (en) z_1. We find that (Ons vind dat)

$$f(x, y, z) = x_1 \oplus y_1 \oplus z_1 \oplus (x_0 + y_0 + z_0).$$

XOR is denoted by "$\oplus$" and OR is denoted by "+". (XOR word as "$\oplus$" geskryf en OR word as "+" geskryf.) We use the mapping (Ons definieer die funksie)

$$f_1(0) = I \qquad f_1(1) = U_H$$

Thus we can map from the triple (x_0, y_0, z_0) to linear operators acting on three qubits (Definieer die funksie van die drietal (x_0, y_0, z_0) na lineêre transformasies):

$$f_3(x_0, y_0, z_0) = f_1(x_0) \otimes f_1(y_0) \otimes f_1(z_0).$$

Let (Laat)

$$|\psi\rangle := \frac{1}{2}(|000\rangle - |011\rangle - |101\rangle - |110\rangle).$$

For each triple (Vir elk van die) (x_0, y_0, z_0) found in the first part of the question, calculate (wat in die eerste deel gevind is, bereken)

$$|\phi\rangle := f_3(x_0, y_0, z_0)|\psi\rangle.$$

Let (laat) s_x, s_y, s_z denote the result (0 or 1) of measuring the first, second and third qubit of $|\phi\rangle$ respectively in the computational basis (die resultaat wees wanneer die eerste, twede en derde qubit van $|\phi\rangle$ in die basis van berekening gemeet word). In each case determine (Vir elke geval bepaal)

$$s_x + s_y + s_z \ mod\ 2, \qquad x_0 + y_0 + z_0.$$

2) Consider the following (Beskou die volgende)

$$|\psi\rangle := a|0\rangle + b|1\rangle, \quad |a|^2 + |b|^2 = 1$$

$$|\phi\rangle := |\psi\rangle \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Show that (Toon aan dat) $|\phi\rangle$ can be written as (kan soos volg geskryf word)

$$\begin{aligned} |\phi\rangle &= \frac{1}{2\sqrt{2}}(|00\rangle + |11\rangle) \otimes (a|0\rangle + b|1\rangle) + \frac{1}{2\sqrt{2}}(|00\rangle - |11\rangle) \otimes (a|0\rangle - b|1\rangle) \\ &+ \frac{1}{2\sqrt{2}}(|01\rangle + |10\rangle) \otimes (a|1\rangle + b|0\rangle) + \frac{1}{2\sqrt{2}}(|01\rangle - |10\rangle) \otimes (a|1\rangle - b|0\rangle) \end{aligned}$$

Describe how measurement of the first two qubits can be used to obtain $|\psi\rangle$ as the last qubit. (Beskryf hoe meting van die eerste twee qubits gebruik kan word om $|\psi\rangle$ vir die laaste qubit te vind.)