TEST VII (TOETS VII) 3B

We use (Ons maak gebruik van) $\{|0\rangle, |1\rangle\}$ as an orthonormal basis for a 2-dimensional Hilbert space in the following questions (vir 'n ortonormale basis vir 'n 2-dimensionele Hilbertruimte in die volgende vrae).

1) Let (laat)

$$P(\theta) := e^{i\theta}|0\rangle\langle 0| + e^{-i\theta}|1\rangle\langle 1| = e^{i\theta}(|0\rangle\langle 0| + e^{-i2\theta}|1\rangle\langle 1|)$$

denote the phase change transform on a single qubit (die fase verandering op een qubit wees). Calculate (Bereken)

$$|s(\theta, \phi)\rangle := P\left(\frac{\pi}{4} - \frac{\phi}{2}\right) U_H P\left(\frac{\theta}{2}\right) U_H |0\rangle.$$

Determine the probability that (Bereken die waarskynlikheid dat) $|s(\theta, \phi)\rangle$ is in the state (word in die toestand gevind)

$$(a)\ |0\rangle \qquad (b)\ |1\rangle \qquad (c)\ |s(\theta', \phi')\rangle.$$

The real parameters θ and ϕ can be interpreted as spherical co-ordinates which define any qubit on the unit sphere called the *Bloch sphere* (Die reële parameters θ en ϕ kan as sferiese koördinate gebruik word wat enige qubit definieer op die eenheid sfeer wat die *Bloch sfeer* genoem word).

2) Find all (Vind alle) $x_A, x_B, x_C \in \{0, 1\}$ such that (sodat) $x_A + x_B + x_C = 1 \ mod\ 2$. We use the mapping (Ons definieer die funksie)

$$f_1(0) = U_H \qquad f_1(1) = I$$

Thus we can map from the triple (x_A, x_B, x_C) to linear operators acting on three qubits (Definieer die funksie van die drietal (x_A, x_B, x_C) na lineêre transformasies):

$$f_3(x_A, x_B, x_C) = f_1(x_A) \otimes f_1(x_B) \otimes f_1(x_C).$$

Let (Laat)

$$|\psi\rangle := \frac{1}{2}(|001\rangle + |010\rangle + |100\rangle - |111\rangle).$$

For each triple (Vir elk van die) (x_A, x_B, x_C) found in the first part of the question, calculate (wat in die eerste deel gevind is, bereken)

$$|\phi\rangle := f_3(x_A, x_B, x_C)|\psi\rangle.$$

Let (laat) s_A, s_B, s_C denote the result (0 or 1) of measuring the first, second and third qubit of $|\phi\rangle$ respectively in the computational basis (die resultaat wees wanneer die eerste, twede en derde qubit van $|\phi\rangle$ in die basis van berekening gemeet word). In each case determine (Vir elke geval bepaal)

$$s_A + s_B + s_C \; mod \; 2, \qquad x_A \cdot x_B \cdot x_C.$$