TEST VI (TOETS VI) 3B

We use (Ons maak gebruik van) $\{|0\rangle, |1\rangle\}$ as an orthonormal basis for a 2-dimensional Hilbert space in the following questions (vir 'n ortonormale basis vir 'n 2-dimensionele Hilbertruimte in die volgende vrae).

1) Calculate (Bereken)

$$\exp\left(tx^2 \frac{d}{dx}\right) x \,.$$

All terms must be summed up. Alle terme moet gesommeer wees. Describe the connection with the solution of the differential equation (Beskryf die verbintenis met die oplossing van die differensiaal vergelyking)

$$\frac{dx}{dt} = x^2, \qquad x(t=0) = x_0$$

2) Let (laat) $A = |0\rangle\langle 0| - |1\rangle\langle 1|$. Calculate (Bereken)

$$U_H A U_H |0\rangle, \qquad U_H A U_H |1\rangle,$$

where (waar) U_H is the (is die) Walsh-Hadamard transform. Calculate (Bereken)

$$(U_H \otimes U_H) U_{CNOT} (U_H \otimes U_H) |a, b\rangle,$$

where (waar) $a, b \in \{0, 1\}$, your answer must be in the form of a ket (die antwoord moet in die vorm van 'n ket wees) $|c, d\rangle$ where (waar) $c, d \in \{0, 1\}$. Remember (Onthou dat)

$$U_H |k\rangle = \frac{1}{\sqrt{2}}(|0\rangle + (-1)^k |1\rangle), \qquad k \in \{0, 1\}.$$

3) Find (Vind)

$$\left[\, |0\rangle\langle 1| \,,\, |1\rangle\langle 0| \,\right]$$

where (waar) $[A, B] = AB - BA$. Determine (Bepaal)

$$\exp(t|0\rangle\langle 1|)$$

$$\exp(t|1\rangle\langle 0|)$$

$$\exp(t|0\rangle\langle 1|)\exp(t|1\rangle\langle 0|)$$

$$\exp(t(|0\rangle\langle 1| + |1\rangle\langle 0|)).$$

4) Determine if the following vectors in $\mathbf{C}^2 \otimes \mathbf{C}^2$ are entangled (Bepaal of verstrikking in die volgende vektore in $\mathbf{C}^2 \otimes \mathbf{C}^2$ voorkom)

$$\frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ -1 \\ -1 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} i \\ 1 \\ -i \\ i \end{pmatrix}.$$