TEST X (TOETS X) 3B

1. Straight forward calculation yields

$$\begin{aligned}
a) &\quad XZ = -Y \\
b) &\quad ZX = Y \\
c) &\quad U_{CNOT}(X \otimes I)U_{CNOT} = X \otimes X \\
d) &\quad U_{CNOT}(I \otimes X)U_{CNOT} = I \otimes X \\
e) &\quad U_{CNOT}(Z \otimes I)U_{CNOT} = Z \otimes I \\
f) &\quad U_{CNOT}(I \otimes Z)U_{CNOT} = Z \otimes Z \\
g) &\quad U_{CNOT}(X \otimes X)U_{CNOT} = X \otimes I \\
h) &\quad U_{CNOT}(Z \otimes Z)U_{CNOT} = I \otimes Z \\
i) &\quad U_{CNOT}U_{CNOT} = I \otimes I
\end{aligned}$$

2.

$$\begin{aligned}
|\phi\rangle &= \frac{1}{\sqrt{2}^n} \sum_{j=0}^{2^n-1} (-1)^{a_j+b_j} |j\rangle \otimes |j\rangle \\
\left(\bigotimes_n U_H\right) \otimes \left(\bigotimes_n U_H\right) |\phi\rangle &= \frac{1}{(2\sqrt{2})^n} \sum_{j=0}^{2^n-1} \sum_{k=0}^{2^n-1} \sum_{l=0}^{2^n-1} (-1)^{a_j+b_j+j*k+j*l} |k\rangle \otimes |l\rangle
\end{aligned}$$

For the case (a) we have

$$\sum_{j=0}^{2^n-1} (-1)^{a_j+b_j+j*k+j*l} = \sum_{j=0}^{2^n-1} (-1)^{j*k+j*l} = 2^n \delta_{kl}$$

For the case (b) we have (for $k = l$ given)

$$\sum_{j=0}^{2^n-1} (-1)^{a_j+b_j+j*k+j*l} = \sum_{j=0}^{2^n-1} (-1)^{a_j+b_j} = 0$$

Thus if condition (a) holds measuring the $2n$ qubits in the computational basis always yields j and j, i.e. the first n qubits always yield exactly the same result as the second n qubits.

If condition b holds then measuring the $2n$ qubits in the computational basis yields $j \neq k$, i.e. the first n qubits never yield the same result as the second n qubits.