Applied Mathematics 3B
Toegepaste Wiskunde 3B
Semester Test 2
Semester Toets 2
9 October 2001

Time: 90 Minutes
Answer all the questions

Tyd: 90 Minute
Beantwoord al die vrae

Question/Vraag 1

Consider

Beskou

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

Find

Vind

$$\exp(-i\sigma_x t/\hbar)\,.$$

Let

Laat

$$\psi(t=0) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Find

Vind

$$\begin{aligned} \psi(t) &= \exp(-i\sigma_x t/\hbar)\psi(t=0) \\ \sigma_y(t) &= \exp(i\sigma_x t/\hbar)\sigma_y \exp(-i\sigma_x t/\hbar) \end{aligned}$$

$$\langle \psi(t), \sigma_y \psi(t) \rangle$$
$$\langle \psi(0), \sigma_y(t)\psi(0) \rangle$$

Question/Vraag 2

Consider the single qubit state

Beskou die een qubit toestand

$$|\psi\rangle := a|0\rangle + b|1\rangle, \qquad |a|^2 + |b|^2 = 1.$$

Rewrite the first two qubits of the state

Herskryf die eerste twee qubits van die staat

$$|\psi\rangle \otimes \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

in terms of the Bell basis, i.e. expand the state of the first two qubits with respect to

in terme van die Bell basis, dus brei die toestand van die eerste twee qubits in terme van

$$\begin{aligned}
|\Phi^+\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \\
|\Phi^-\rangle &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle), \\
|\Psi^+\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), \\
|\Psi^-\rangle &= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle).
\end{aligned}$$

Describe how to obtain $|\psi\rangle$ as the state of the last qubit by measuring the first two qubits in the Bell basis.

uit. Beskryf hoe om $|\psi\rangle$ as die staat van die laaste qubit te vind deur meting van die eerste twee qubits met betrekking tot die Bell basis.

Question/Vraag 3

Show that

$$|0_H\rangle := \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$|1_H\rangle := \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

forms an orthonormal basis for $\mathbf{C}^2$. What is the probability that $|0\rangle$ is in the state
a) $|0_H\rangle$
b) $|1_H\rangle$
Thus determine how to obtain $|0_H\rangle$ and $|1_H\rangle$ using only measurement and the phase change operation

Toon aan dat

'n ortonormale basis vir $\mathbf{C}^2$ vorm. Bereken die waarskynlikheid dat $|0\rangle$ in die toestand
a) $|0_H\rangle$
b) $|1_H\rangle$
Bepaal hoe om die toestand $|0_H\rangle$ en $|1_H\rangle$ te skep deur net van meting en die fase verandering

$$U_{PS} := |0\rangle\langle 0| - |1\rangle\langle 1|$$

Let

gebruik te maak. Laat

$$f : \{0,1\} \to \{0,1\}$$
$$U_f := |0f(0)\rangle\langle 00| + |0\overline{f(0)}\rangle\langle 01| + |1f(1)\rangle\langle 10| + |1\overline{f(1)}\rangle\langle 11|$$

Determine in terms of $|0_H\rangle$ and $|1_H\rangle$

Bepaal in terme van $|0_H\rangle$ en $|1_H\rangle$

$c)$ $\quad U_f|0_H\rangle \otimes |0_H\rangle$
$d)$ $\quad U_f|0_H\rangle \otimes |1_H\rangle$

Question/Vraag 4

Determine when each of the following elements of $\mathbf{C}^4$ are entangled

Bepaal wanneer elk van die volgende elemente van $\mathbf{C}^4$ verstrik is

$$a) \quad \frac{1}{\sqrt{2}}(\sin\theta, \sin\phi, \cos\phi, \cos\theta)^T$$

$$b) \quad \frac{1}{\sqrt{3}}(1, \sin\theta, 1, \cos\theta)^T$$

$$c) \quad \frac{1}{\sqrt{2}}(U_H \otimes U_H)(1, 0, 0, 1)^T$$

$$d) \quad \frac{1}{\sqrt{2}}U_{CNOT}(1, 0, 0, 1)^T$$

where

waar

$$U_H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

and $\otimes$ denotes the Kronecker product.

en $\otimes$ is die Kronecker produk.