Applied Mathematics 3B

Toegepaste Wiskunde 3B

Semester Test 1

Semester Toets 1

11 September 2001

Time: 90 Minutes
Answer all the questions

Tyd: 90 Minute
Beantwoord al die vrae

Question/Vraag 1

We consider the Hilbert space $L_2[-\frac{1}{2},\frac{1}{2}]$. Show that the following basis

Beskou die Hilbertruimte $L_2[-\frac{1}{2},\frac{1}{2}]$. Toon aan dat elk van die volgende basis

$$F_1 = \{\phi_k(x) = \exp(2\pi ikx),\ k \in \mathbf{Z}\}$$

$$F_2 = \{\psi_k(x) = \sqrt{2}\sin(2\pi kx),\ k \in \mathbf{N}\}$$

each form an orthonormal basis in the Hilbert space. Expand the step function

'n ortonormale basis in die Hilbertruimte is. Brei die funksie

$$f(x) = \begin{cases} -1 & x \in [-\frac{1}{2}, 0] \\ 1 & x \in (0, \frac{1}{2}] \end{cases}$$

with respect to the basis F_1 and with respect to the basis F_2. Show that the two expansions are equivalent. Remember

met betrekking tot die basis F_1 en die basis F_2 uit. Toon aan dat die twee uitbreidings ekwivalent is. Onthou dat

$$2\sin(x)\sin(y) = \cos(x-y) - \cos(x+y).$$

Question/Vraag 2

Consider the Hamilton operator

Beskou die Hamiltonoperator

$$\hat{S}_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} .$$

Find

Vind

$$\exp(-i\hat{S}_y t/\hbar) .$$

Let

Laat

$$\psi(t=0) = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} .$$

Find

Vind

$$\psi(t) = \exp(-i\hat{S}_y t/\hbar)\psi(t=0) .$$

Calculate the probability to find the particle in the initial state after time t.

Bereken die waarskynlikheid om die partikel in die aanvangsstaat na tyd t te vind.

Question/Vraag 3

Calculate the eigenvalues and corresponding normalized eigenvectors of the following matrices operating on $\mathbf{C}^2$

Bepaal die eiewaardes en oorstemmende genormaliseerde eievektore van die volgende matrikse wat op $\mathbf{C}^2$ werk

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The matrices represent observables. Determine the probability that measurement, with respect to each observable, of a system in the state

Die matrikse is voorstellings van waarneembare eienskappe. Bereken die waarskynlikheid dat die resultaat van meting, met betrekking tot elke eienskap, van 'n stelsel in die staat

$$\frac{1}{\sqrt{2}}(1, -1)^T$$

will yield the result corresponding to each of the eigenvalues determined previously (calculate for each eigenvalue).

ooreenstem met die eiewaardes wat alreeds bereken is (bereken vir elke eiewaarde).

Question/Vraag 4

Let $\{|0\rangle, |1\rangle\}$ be an orthonormal basis for a two dimensional Hilbert space, and

Laat $\{|0\rangle, |1\rangle\}$ 'n ortonormale basis vir 'n twee dimensionele Hilbertruimte wees, en

$$U_H|k\rangle := \frac{1}{\sqrt{2}}\Big(|0\rangle + (-1)^k|1\rangle\Big), \quad k \in \{0,1\}$$

$$U_{PS(\theta)} := |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + e^{i\theta}|11\rangle\langle 11|$$

$$U_{CNOT} := |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 11| + |11\rangle\langle 10|$$

From the definition show that

Toon van die definisie aan dat

$$U_H U_H = I.$$

Calculate

Bereken

$$(I \otimes U_H)U_{PS(\pi)}(I \otimes U_H)|ab\rangle$$

and

en

$$(I \otimes U_H)U_{CNOT}(I \otimes U_H)|ab\rangle$$

where $a, b \in \{0,1\}$. The answers must be in the form of a ket $|cd\rangle$ where $c, d \in \{0,1\}$. What computations do these transforms implement?

waar $a, b \in \{0,1\}$. Die antwoorde moet in die vorm van 'n ket $|cd\rangle$ waar $c, d \in \{0,1\}$ gegee word. Wat word deur hierdie transformasies bereken?