

TWK2A Dirac delta function (Section 7.5) Solutions

1.

$$\begin{aligned}y' - 3y &= \delta(t - 2) \\ \Rightarrow sY(s) - y(0) - 3Y(s) &= e^{-2s} \\ \Rightarrow Y(s) &= \frac{e^{-2s}}{s - 3} \\ \Rightarrow y(t) &= e^{3t-6}\mathcal{U}(t - 2).\end{aligned}$$

2.

$$\begin{aligned}y' + y &= \delta(t - 1) \\ \Rightarrow sY(s) - y(0) + Y(s) &= e^{-s} \\ \Rightarrow Y(s) &= \frac{e^{-s}}{s + 1} + \frac{2}{s + 1} \\ \Rightarrow y(t) &= e^{-t+1}\mathcal{U}(t - 1) + 2e^{-t}.\end{aligned}$$

3.

$$\begin{aligned}y'' + y &= \delta(t - 2\pi) \\ \Rightarrow s^2Y(s) - sy(0) - y'(0) + Y(s) &= e^{-2\pi s} \\ \Rightarrow Y(s) &= \frac{e^{-2\pi s}}{s^2 + 1} + \frac{1}{s^2 + 1} \\ \Rightarrow y(t) &= \sin t + \sin(t - 2\pi)\mathcal{U}(t - 2\pi) \\ \Rightarrow y(t) &= \sin t + \sin t\mathcal{U}(t - 2\pi) \\ &= (1 + \mathcal{U}(t - 2\pi))\sin t.\end{aligned}$$

4.

$$\begin{aligned}y'' + 16y &= \delta(t - 2\pi) \\ \Rightarrow s^2 Y(s) - sy(0) - y'(0) + 16Y(s) &= e^{-2\pi s} \\ \Rightarrow Y(s) &= \frac{e^{-2\pi s}}{s^2 + 16} \\ \Rightarrow y(t) &= \frac{1}{4} \sin(4(t - 2\pi)) \mathcal{U}(t - 2\pi) \\ \Rightarrow y(t) &= \frac{1}{4} \sin(4t - 8\pi) \mathcal{U}(t - 2\pi) \\ &= \frac{1}{4} \sin(4t) \mathcal{U}(t - 2\pi).\end{aligned}$$

5.

$$\begin{aligned}y'' + y &= \delta(t - 2\pi) + \delta(t - 4\pi) \\ \Rightarrow s^2 Y(s) - sy(0) - y'(0) + Y(s) &= e^{-2\pi s} + e^{-4\pi s} \\ \Rightarrow Y(s) &= \frac{e^{-2\pi s}}{s^2 + 1} + \frac{e^{-4\pi s}}{s^2 + 1} + \frac{s}{s^2 + 1} \\ \Rightarrow y(t) &= \sin(t - 2\pi) \mathcal{U}(t - 2\pi) + \sin(t - 4\pi) \mathcal{U}(t - 4\pi) + \cos t \\ \Rightarrow y(t) &= (\mathcal{U}(t - 2\pi) + \mathcal{U}(t - 4\pi)) \sin t + \cos t.\end{aligned}$$