# TWK2A
# Solutions by substitutions
# Problems

1. Solve the homogeneous DE $ydx = 2\left(x + y\right)dy$.
2. Solve the homogeneous DE $\frac{dy}{dx} = \frac{x+3y}{3x+y}$.
3. Solve the IVP $\left(x + ye^{\frac{y}{x}}\right)dx - xe^{\frac{y}{x}}dy = 0; y(1) = 0$.
4. Solve the Bernoulli equation $\frac{dy}{dx} - y = e^x y^2$.
5. Solve the IVP $\sqrt{y}\frac{dy}{dx} + \sqrt{y^3} = 1; y(0) = 4$.
6. Solve $y' = 1 + e^{y-x+5}$ using the substitution $u = y - x + 5$.
7. The DE $y' = P(x) + Q(x)y + R(x)y^2$ is known as Ricatti's equation. Such a DE may be solved by a succession of two substitutions provided we know a particular solution $y_1$ of the DE. Use the substitution $y = y_1 + u$ and then discuss what to do next. Hence, find a one-parameter family of solutions for the DE
$$y' = -\frac{4}{x^2} - \frac{1}{x}y + y^2$$
which has the particular solution $y_1 = \frac{2}{x}$.
8. Solve the homogeneous DE $xy' - y = \sqrt{x^2 + y^2}$.
9. Solve the IVP $ydx + x\left(\ln x - \ln y - 1\right)dy = 0; y(1) = e$.
10. Show that the DE $\left(x + y\right)dx + xdy = 0$ is homogeneous, and then solve it.
11. Solve the Bernoulli equation $3\left(1 + t^2\right)\frac{dy}{dt} = 2ty\left(y^3 - 1\right)$.
12. Show that the DE $\left(y^2 + yx\right)dx - x^2dy = 0$ is homogeneous, and then solve it.
13. Show that the DE $x\frac{dy}{dx} = y + \sqrt{x^2 - y^2}, x > 0$ is homogeneous, and then solve it.
14. Solve the DE $\frac{dy}{dx} = \frac{1-x-y}{x+y}$ using an appropriate substitution.

15. Show that the DE $-ydx + \left(x + \sqrt{xy}\right) dy = 0$ is homogeneous, and then solve it.

16. Solve $x\frac{dy}{dx} + y = \frac{1}{y^2}$.

17. Solve $x^2\frac{dy}{dx} - 2xy = 3y^4, \;\; y\left(1\right) = \frac{1}{2}$.