

TWK2A Linear DEs Problems

1. Find the general solution of $3y' + 12y = 4$. Give the largest interval on which the general solution is defined.
2. Find the general solution of $xy' + (1 + x)y = e^{-x} \sin 2x$. Give the largest interval on which the general solution is defined.
3. Find the general solution of $(x + 2)^2 y' = 5 - 8y - 4xy$. Give the largest interval on which the general solution is defined.
4. Solve $y \frac{dx}{dy} - x = 2y^2$ with $y(1) = 5$. Give the largest interval on which the solution is defined.
5. Find the general solution of $x^2 y' + xy = 1$. Give the largest interval on which the general solution is defined.
6. Find the general solution of $x^2 y' + x(x + 2)y = e^x$. Give the largest interval on which the general solution is defined.
7. Find the general solution of $xy' + (3x + 1)y = e^{-3x}$. Give the largest interval on which the general solution is defined.
8. Solve $(x + 1)y' + y = \ln x$ with $y(1) = 10$. Give the largest interval on which the solution is defined.
9. Find the general solution of $y' + 2xy = x^3$. Give the largest interval on which the general solution is defined.
10. Find the general solution of $ydx = (ye^y - 2x)dy$. Give the largest interval on which the general solution is defined.
11. Find the general solution of $(x + 1)y' + (x + 2)y = 2xe^{-x}$. Give the largest interval on which the general solution is defined.
12. Solve $y' + (\tan x)y = \cos^2 x$ with $y(0) = -1$. Give the largest interval on which the solution is defined.

13. In determining the integrating factor for a linear DE, we do not use a constant in the evaluation of $\int P dx$. Explain why this does not affect the solution.

14. Consider the system

$$\frac{dx}{dt} = -\lambda_1 x \qquad \frac{dy}{dt} = \lambda_1 x - \lambda_2 y$$

where λ_1 and λ_2 are constants. Discuss how to solve this system subject to $x(0) = x_0, y(0) = y_0$. Implement your ideas.

15. Find the general solution of $y' + y = e^{-3x}$. Give the largest interval on which the general solution is defined.

16. Find the general solution of $y' + 3x^2 y = 10x^2$. Give the largest interval on which the general solution is defined.

17. Find the general solution of $(\cos x) y' + (\sin x) y = 1$. Give the largest interval on which the general solution is defined.

18. Find the general solution of $(x^2 - 1) y' + 2y = (x + 1)^2$. Give the largest interval on which the general solution is defined.