TWK2A
Exact DEs
Problems

1. Show that the DE $(\sin y - y\sin x)\,dx + (\cos x + x\cos y - y)\,dy = 0$ is exact and, hence, solve the DE.

2. Show that the DE $\left(1 + \ln x + \frac{y}{x}\right)dx - (1 - \ln x)\,dy = 0$ is exact and, hence, solve the DE.

3. Show that the DE $(4t^3y - 15t^2 - y)\,dt + (t^4 + 3y^2 - t)\,dy = 0$ is exact and, hence, solve the DE.

4. Solve the IVP $(x+y)^2\,dx + (2xy + x^2 - 1)\,dy = 0$ with $y(1) = 1$.

5. The DE $\left(x - \sqrt{x^2+y^2}\right)dx + ydy = 0$ is not exact. Show how the rearrangement $\frac{(xdx+ydy)}{\sqrt{x^2+y^2}} = dx$ and the observation $\frac{1}{2}d\left(x^2+y^2\right) = xdx + ydy$ can lead to a solution.

6. Show that every separable first-order DE is also exact.

7. Show that the DE $\left(1 - \frac{3}{y} + x\right)\frac{dy}{dx} + y = \frac{3}{x} - 1$ is exact and, hence, solve the DE.

8. Solve the IVP $(e^x + y)\,dx + (2 + x + ye^y)\,dy = 0$ with $y(0) = 1$.

9. Solve $y(x+y+1)\,dx + (x+2y)\,dy = 0$ by finding an integrating factor that makes the DE exact.

10. Solve $\cos xdx + \left(1 + \frac{2}{y}\right)\sin xdy = 0$ by finding an integrating factor that makes the DE exact.

11. Solve $(x^2 + y^2 - 5)\,dx = (y + xy)\,dy$ with $y(0) = 1$, by finding an integrating factor that makes the DE exact.

12. Show that the DE $(\tan x - \sin x\sin y)\,dx + \cos x\cos ydy = 0$ is exact and, hence, solve the DE.

13. Solve the IVP $(y^2 \cos x - 3x^2 y - 2x)\, dx + (2y \sin x - x^3 + \ln y)\, dy = 0$ with $y(0) = e$.

14. Show that the DE $\left(2y - \frac{1}{x} + \cos 3x\right) \frac{dy}{dx} + \frac{y}{x^2} - 4x^3 + 3y \sin 3x = 0$ is not exact.

15. Solve the IVP $\left(\frac{3y^2 - t^2}{y^5}\right) \frac{dy}{dt} + \frac{t}{2y^4} = 0$ with $y(1) = 1$.

16. Verify that $(x^2 + 2xy - y^2)\, dx + (y^2 + 2xy - x^2)\, dy = 0$ is not exact. Multiply the DE by the integrating factor $\mu(x, y) = (x + y)^{-2}$ and verify that the resulting DE is exact.

17. Show that the DE $(2x + 4)\, dx + (3y - 8)\, dy = 0$ is exact and, hence, solve the DE.

18. Show that the DE $(x^3 + y^3)\, dx + 3xy^2 dy = 0$ is exact and, hence, solve the DE.

19. Solve the initial-value problem $(4y + 2t - 5)\, dt + (6y + 4t - 1)\, dy = 0$ with $y(-1) = 0$.

20. Find the value of k such that the DE $(y^3 + kxy^4 - 2x)\, dx + (3xy^2 + 20x^2 y^3)\, dy = 0$ is exact. Hence, solve the equation.

21. Prove that, in general, the linear DE

$$y' + P(x)y = f(x)$$

is not exact. Then prove that when this DE is multiplied through by the integrating factor $e^{\int P dx}$, the resulting DE is exact.