

TWK2A

Chapter 1

Problems

1. Is $(y^2 - 1)dx + xdy = 0$ linear or nonlinear? What is its order?
2. Is $(\sin \theta)y''' - (\cos \theta)y' = 2$ linear or nonlinear? What is its order?
3. Is $x'' - \left(1 - \frac{(x')^2}{3}\right)x' + x = 0$ linear or nonlinear? What is its order?
4. Verify that $y(x) = -\cos x \ln(\sec x + \tan x)$ is a solution of $y'' + y = \tan x$.
5. Verify that $\phi_1(x) = x^2$ and $\phi_2(x) = -x^2$ are solutions of $xy' - 2y = 0$ on the interval $(-\infty, \infty)$.
6. Verify that $x(t) = e^{-2t} + 3e^{6t}$ and $y(t) = -e^{-2t} + 5e^{6t}$ is a solution of the system $x' = x + 3y; y' = 5x + 3y$. Here, x and y are dependent variables and the differentiation is wrt the independent variable t .
7. Is $xy''' - (y')^4 + y = 0$ linear or nonlinear? What is its order?
8. Is $udv + (v + uv - ue^u)du = 0$ linear or nonlinear? What is its order?
9. Is $\frac{d^2R}{dt^2} = -\frac{k}{R^2}$ linear or nonlinear? What is its order?
10. Verify that $y(x) = e^{3x} \cos 2x$ is a solution of $y'' - 6y' + 13y = 0$.
11. Show that $x + y + e^{xy} = 0$ is an implicit solution of $(1 + xe^{xy}) \frac{dy}{dx} + 1 + ye^{xy} = 0$.
12. Is $t^5y^{(4)} - t^3y'' + 6y = 0$ linear or nonlinear? What is its order?
13. Is $\frac{d^2u}{dr^2} + \frac{du}{dr} + u = \cos(r + u)$ linear or nonlinear? What is its order?
14. Verify that the piecewise-defined function

$$y = \begin{cases} -x^2 & x < 0 \\ x^2 & x \geq 0 \end{cases}$$

is a solution of $xy' - 2y = 0$ on $(-\infty, \infty)$.

15. Verify that the one-parameter family $y^2 - 2y = x^2 - x + c$ is an implicit solution of $(2y - 2)y' = 2x - 1$. Hence, find an implicit solution of the DE that satisfies $y(0) = 1$.
16. Verify that $y(x) = \sin(\ln x)$ is a particular solution of $x^2y'' + xy' + y = 0$.
17. Is $xy'' - \sqrt{y'} + y = 0$ linear or nonlinear? What is its order?
18. Is the equation $2y\frac{dy}{dx} + 2x = 0$ linear in y ? Is the equation linear in x ? Find $u(x, y)$ such that the equation can be rewritten as a linear equation in u .
19. Consider the differential equation $y' = e^{-x^2}$. Explain why a solution of the DE must be an increasing function on any interval of the x -axis. What are $\lim_{x \rightarrow -\infty} y'$ and $\lim_{x \rightarrow \infty} y'$? What does this suggest about a solution as $x \rightarrow \pm\infty$?
20. Consider the differential equation $\frac{dy}{dx} = y(a - by)$, where a and b are positive constants. Find two constant solutions of this DE.
21. Consider the differential equation $y' = y^2 + 4$. Explain why has DE has no constant real-valued solutions.
22. Verify that the three-parameter family $y(x) = c_1x^{-1} + c_2x + c_3x \ln x + 4x^2$ is a solution of $x^3y''' + 2x^2y'' - xy' + y = 12x^2$.
23. Explain why the piecewise-defined function

$$y = \begin{cases} \sqrt{25 - x^2} & -5 < x < 0 \\ -\sqrt{25 - x^2} & 0 \leq x < 5 \end{cases}$$

is not a solution of $y' = -x/y$ on the interval $(-5, 5)$.

24. Find values of m such that the function $y = x^m$ is a solution of $x^2y'' - 7xy' + 15y = 0$.
25. The differential equation $y'' - y = 0$ has the two-parameter family of solutions $y = c_1e^x + c_2e^{-x}$. Determine the parameters using the initial conditions $y(1) = 0$ and $y'(1) = e$.
26. The differential equation $y'' - y = 0$ has the two-parameter family of solutions $y = c_1e^x + c_2e^{-x}$. Determine the parameters using the initial conditions $y(0) = 0$ and $y'(0) = 0$.

27. Show that $\cos(x - y) = c$ is an implicit solution of $\tan(x - y) \frac{d^2y}{dx^2} = \left(1 - \frac{dy}{dx}\right)^2$ and hence solve the IVP $y(1) = 1$.