

TWK2A

Second translation theorem (Section 7.3.2)

Problems

1. Determine

$$\mathcal{L}\{e^{2-t}\mathcal{U}(t-2)\}.$$

2. Determine

$$\mathcal{L}\left\{\sin t\mathcal{U}\left(t-\frac{\pi}{2}\right)\right\}.$$

3. Determine

$$\mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{s^2+1}\right\}.$$

4. Determine

$$\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^2(s-1)}\right\}.$$

5. Determine $\mathcal{L}\{f(t)\}$, where

$$f(t) = \begin{cases} 1, & 0 \leq t < 4 \\ 0, & 4 \leq t < 5 \\ 1, & t \geq 5 \end{cases}.$$

6. Determine

$$\mathcal{L}\{(t-1)\mathcal{U}(t-1)\}.$$

7. Determine

$$\mathcal{L}\{\cos 2t\mathcal{U}(t-\pi)\}.$$

8. Determine

$$\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s^3}\right\}.$$

9. Find the Laplace transform of

$$f(t) = \begin{cases} \sin t, & 0 \leq t < 2\pi \\ 0, & 2\pi \leq t < 3\pi \\ t^2, & 3\pi \leq t < 4\pi \end{cases}.$$

10. Use the Laplace transform to solve the IVP

$$y' + 2y = \begin{cases} t, & 0 \leq t < 1 \\ 0, & t \geq 1 \end{cases}$$

with $y(0) = 0$.

11. Use the Laplace transform to solve the IVP

$$y'' + 4y = \begin{cases} 1, & 0 \leq t < 1 \\ 0, & t \geq 1 \end{cases}$$

with $y(0) = 0, y'(0) = -1$.

12. Use the second translation theorem to determine

$$\mathcal{L}^{-1} \left\{ \frac{s}{(s+7)^3} \right\}.$$

13. Use the second translation theorem to determine

$$\mathcal{L}^{-1} \left\{ \frac{s^2}{(s-3)^4} \right\}.$$