TWK2A
Nonlinear DEs (Section 4.9)
Problems

1. Verify that $y_1 = 1$ and $y_2 = x^2$ are solutions of $yy'' = \frac{1}{2}(y')^2$, but that $y = c_1 y_1 + c_2 y_2$ is, in general, not.
2. Solve $(y+1)\,y'' = (y')^2$.
3. Solve $y'' + 2y\,(y')^3 = 0$.
4. Show that the substitution $u = y'$ in $xy'' = y' + (y')^3$ leads to a Bernoulli equation, and then solve.
5. Solve $y''' = \sqrt{1 + (y'')^2}$.
6. Solve $y'' + (y')^2 + 1 = 0$.