TWK2A
Variation of parameters (Section 4.6)
Problems

Solve the DEs in these exercises using *variation of parameters*.

1. $y'' + y = \sin x$
2. $y'' - y = \cosh\left(\frac{x}{2}\right)$
3. $4y'' - y = xe^{x/2}$ with $y(0) = 1, y'(0) = 0$.
4. Given that $y_1 = x^{-1/2}\cos x$ and $y_2 = x^{-1/2}\sin x$ are linearly independent solutions of $x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$ on $(0, \infty)$, find the general solution of $x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = x^{3/2}$ on $(0, \infty)$.
5. Discuss how the methods of undetermined coefficients and variation of parameters may be combined to solve $y'' - 2y' + y = 4x^2 - 3 + x^{-1}e^x$. Hence, solve the DE.
6. $y'' + y = \sec\theta\tan\theta$
7. $y'' - 9y = 9xe^{-3x}$
8. $2y'' + y' - y = x + 1$ with $y(0) = 1, y'(0) = 0$.
9. $y''' + 4y' = \sec 2x$
10. $y'' + y = \tan x$
11. $y'' - 4y' + 4y = (12x^2 - 6x)e^{2x}$ with $y(0) = 1, y'(0) = 0$.
12. $y'' - 4y = x^{-1}e^{2x}$
13. $y'' + 2y' + y = e^{-x}\ln x$
14. $y'' + y = \cos^2 x$
15. $y'' + 3y' + 2y = \sin(e^x)$
16. $x^2y'' + xy' + y = \sec(\ln x)$

17. $3y'' - 6y' + 30y = 15\sin x + e^x \tan 3x$

18. Solve $x^2y'' + xy' - 4y = x^{-2}$ given that $y_1 = x^2$ is a solution of the associated homogeneous equation on $(0, \infty)$.