

TWK2A

Reduction of order (Section 4.2)

Problems

1. If $y_1 = e^{5x}$ is a solution of $y'' - 25y = 0$, use reduction of order to find a second solution.
2. If $y_1 = x^4$ is a solution of $x^2y'' - 7xy' + 16y = 0$, use reduction of order to find a second solution.
3. If $y_1 = \ln x$ is a solution of $xy'' + y' = 0$, use reduction of order to find a second solution.
4. If $y_1 = x^2 \cos(\ln x)$ is a solution of $x^2y'' - 3xy' + 5y = 0$, use reduction of order to find a second solution.
5. If $y_1 = e^{\frac{x}{3}}$ is a solution of $6y'' + y' - y = 0$, use reduction of order to find a second solution.
6. If $y_1(x)$ is a solution to a linear homogeneous second-order DE, a second solution to the equation is given by

$$y_2(x) = y_1(x) \int \frac{e^{-\int P(x)dx}}{y_1^2(x)} dx,$$

where $P(x)$ is the coefficient of y' in the standard form of the ODE. Show that y_1 and y_2 are linearly independent.

7. Consider the equation

$$ay'' + by' + cy = 0.$$

Given that $y_1(x) = e^{-\frac{b}{2a}x}$ is a solution to this DE, use reduction of order to find a second solution.