TWK2A
Section 4.1
Problems

1. Verify that $y = c_1 e^{4x} + c_2 e^{-x}$ is the general solution of $y'' - 3y' - 4y = 0$ on $(-\infty, \infty)$. Solve the IVP $y(0) = 1, y'(0) = 2$.

2. Verify that $y = c_1 + c_2 \cos x + c_3 \sin x$ is the general solution of $y''' + y' = 0$ on $(-\infty, \infty)$. Solve the IVP $y(\pi) = 0, y'(\pi) = 2, y''(\pi) = -1$.

3. Given that $y = c_1 + c_2 x^2$ is a two-parameter family of solutions of $xy'' - y' = 0$ on $(-\infty, \infty)$, show that constants c_1 and c_2 do not exist which satisfy $y(0) = 0, y'(0) = 1$. Explain why this does not violate Theorem 4.1.

4. The function $y = c_1 e^x \cos x + c_2 e^x \sin x$ is a solution of $y'' - 2y' + 2y = 0$ on $(-\infty, \infty)$. Determine whether or not constants c_1 and c_2 can be found satisfying **(a)** $y(0) = 1, y'(\pi) = 0$, **(b)** $y(0) = 1, y(\pi) = -1$, **(c)** $y(0) = 1, y(\frac{\pi}{2}) = 1$, **(d)** $y(0) = 0, y(\pi) = 0$.

5. Are $f_1(x) = x, f_2(x) = x^2$ and $f_3(x) = 4x - 3x^2$ linearly independent on $(-\infty, \infty)$?

6. Are $f_1(x) = 5, f_2(x) = \cos^2 x$ and $f_3(x) = \sin^2 x$ linearly independent on $(-\infty, \infty)$?

7. Show that $\{x^2, x^3\}$ forms a fundamental set of solutions of $x^2y'' - 6xy' + 12y = 0$ on $(0, \infty)$. What is the general solution?

8. Verify that $y_{p_1} = 3e^{2x}$ and $y_{p_2} = x^2 + 3x$ are, respectively, particular solutions of

$$\begin{aligned} y'' - 6y' + 5y &= -9e^{2x} \\ y'' - 6y' + 5y &= 5x^2 + 3x - 16 \end{aligned}$$

Hence, find particular solutions of

$$y'' - 6y' + 5y = 5x^2 + 3x - 16 - 9e^{2x}$$

and

$$y'' - 6y' + 5y = -10x^2 - 6x + 32 + e^{2x}.$$

9. (a) By inspection find a particular solution of

$$y'' + 2y = 10.$$

(b) By inspection find a particular solution of

$$y'' + 2y = -4x.$$

(c) Find a particular solution of

$$y'' + 2y = -4x + 10.$$

(d) Find a particular solution of

$$y'' + 2y = 8x + 5.$$

10. Suppose that $y_1 = e^x$ and $y_2 = e^{-x}$ are solutuions of a homogeneous linear differential equation. Explain why $y_3 = \cosh x$ and $y_4 = \sinh x$ are also solutions of the equation.

11. The function $y = c_1x^2 + c_2x^4 + 3$ is a solution of $x^2y'' - 5xy' + 8y = 24$ on $(-\infty, \infty)$. Determine whether or not constants c_1 and c_2 can be found satisfying **(a)** $y(-1) = 0, y(1) = 4,$ **(b)** $y(0) = 1, y(1) = 2,$ **(c)** $y(0) = 3, y(1) = 4,$ **(d)** $y(1) = 3, y(2) = 15.$

12. Are $f_1(x) = 0, f_2(x) = x$ and $f_3(x) = e^x$ linearly independent on $(-\infty, \infty)$?

13. Are $f_1(x) = 2 + x$ and $f_2(x) = 2 + |x|$ linearly independent on $(-\infty, \infty)$?

14. Show that $\left\{e^{\frac{x}{2}}, xe^{\frac{x}{2}}\right\}$ forms a fundamental set of solutions of $4y'' - 4y' + y = 0$ on $(-\infty, \infty)$. What is the general solution?

15. Show that $\{\cos(\ln x), \sin(\ln x)\}$ forms a fundamental set of solutions of $x^2y'' + xy' + y = 0$ on $(0, \infty)$. What is the general solution?

16. Verify that $y = c_1 \cos x + c_2 \sin x + x \sin x + (\cos x) \ln(\cos x)$ is the general solution of $y'' + y = \sec x$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

17. Verify that $y = c_1x^{-\frac{1}{2}} + c_2x^{-1} + \frac{1}{15}x^2 - \frac{1}{6}x$ is the general solution of $2x^2y'' + 5xy' + y = x^2 - x$ on $(0, \infty)$.

18. Are $f_1(x) = \cos 2x$, $f_2(x) = 1$ and $f_3(x) = \cos^2 x$ linearly independent on $(-\infty, \infty)$?

19. Suppose that $y_1, y_2, ..., y_k$ are k nontrivial (i.e. $y_j \neq 0$) solutions of a homogeneous linear nth-order DE with constant coefficients, and that $k = n + 1$. Is the set $\{y_1, y_2, ..., y_k\}$ linearly dependent or independent on $(-\infty, \infty)$? Discuss.

20. Use the Wronskian to show that

$$\{e^x, e^{-2x}\}$$

are linearly independent on $(0, \infty)$.

21. Use the Wronskian to show that

$$\{1, \cos x, \sin x\}$$

are linearly independent on $(-\infty, \infty)$. Note that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = iae - afh - idb + dch + gbf - gce.$$