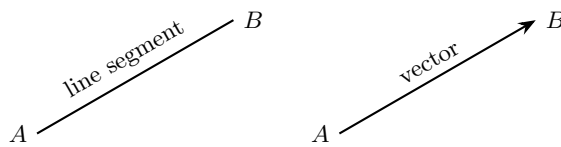
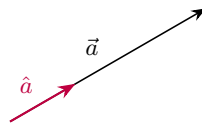


Vectors

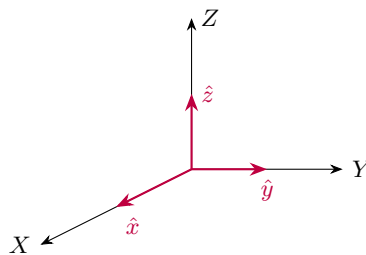
- A *scalar* is a number which is used to represent some quantity, for example magnitudes (which is the *size* of a mathematical object).
- Scalars are indicated by regular Roman text, for example a or x , or by using Greek alphabet letters, for example α or λ .
- A *vector* is a mathematical object with magnitude and direction.
- Geometrically, vectors are indicated by an arrow; vectors may also be represented by a directed line segment, that is, a line segment with a specified *initial point* A and a specified *terminal point* B and indicated as \overline{AB} .



- Algebraically, and in text, vectors are indicated by bold face letters, for example \mathbf{a} ; by a bar over the letter, for example \bar{a} ; by an arrow over the letter, for example \vec{a} .
- The magnitude of a vector \vec{a} is a scalar value and is indicated by writing the vector in regular Roman text, for example a , or by enclosing the vector in two vertical bars, for example $|\vec{a}|$.
- An *unit vector* is a vector which has magnitude equal to one. If \vec{a} is a vector with magnitude $|\vec{a}|$, then the unit vector with the same direction as \vec{a} is indicated by $\hat{a} = \vec{a}/a$.

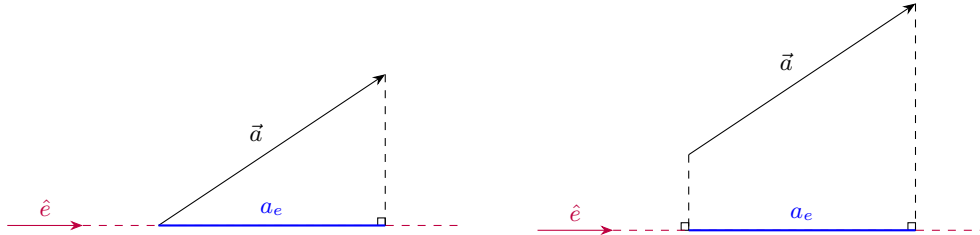


- The unit vectors in the same direction has the positive axes of the Cartesian reference system are indicated by \hat{x} , \hat{y} , and \hat{z} .

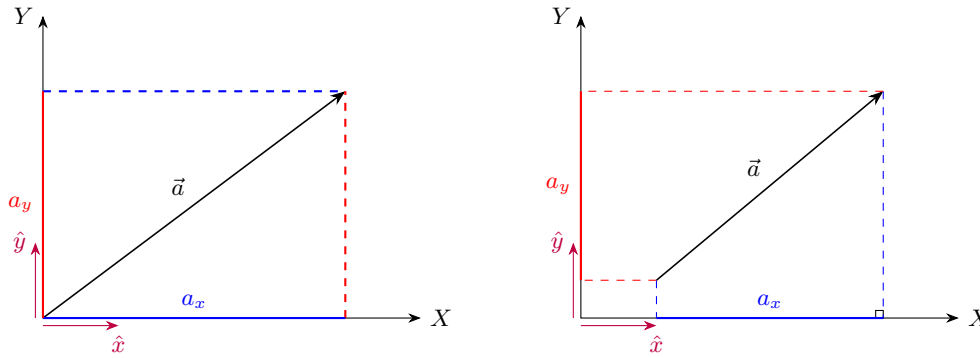


Vector components

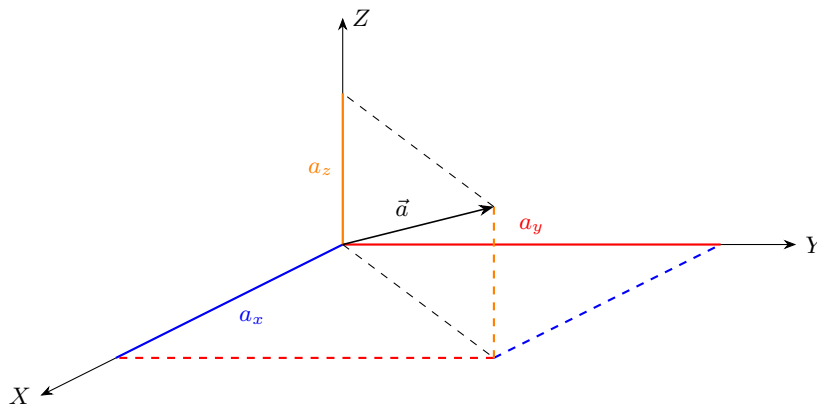
- The *component* of a vector in a given direction can be thought of as the magnitude of the vector measured in that direction.
- In the following figure, a_e is the component of the vector \vec{a} in the e direction.

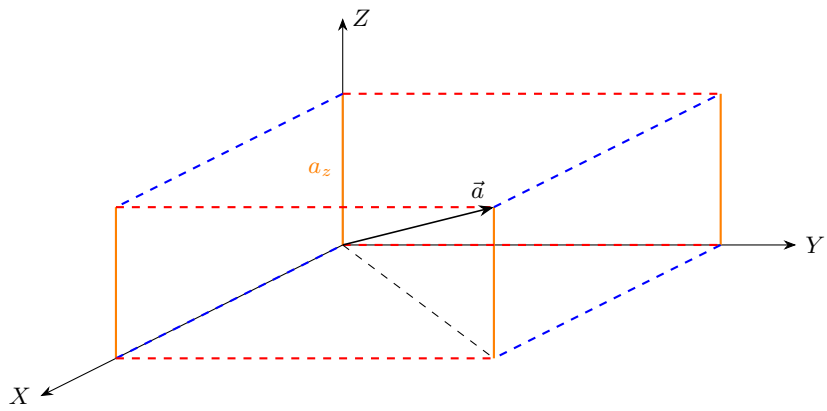
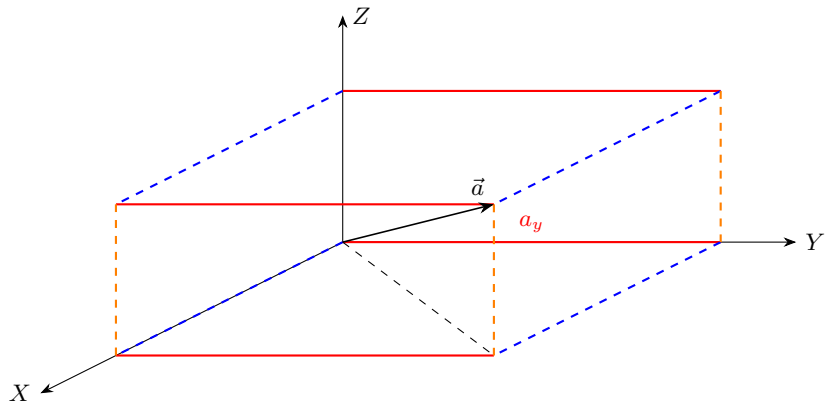
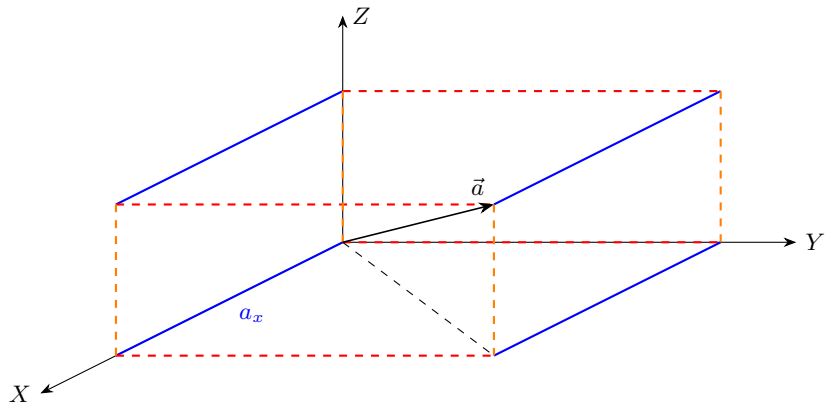


- The components a_x and a_y of a vector \vec{a} in the two-dimensional Cartesian reference system are shown in the following figure:



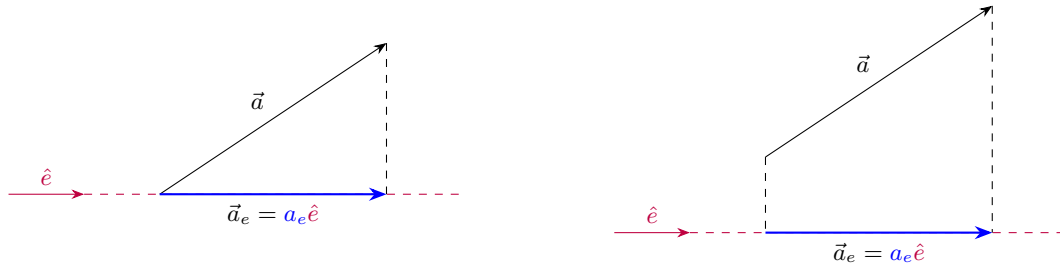
- The components a_x , a_y , and a_z of a vector \vec{a} in the three-dimensional Cartesian reference system are shown in the following figures:



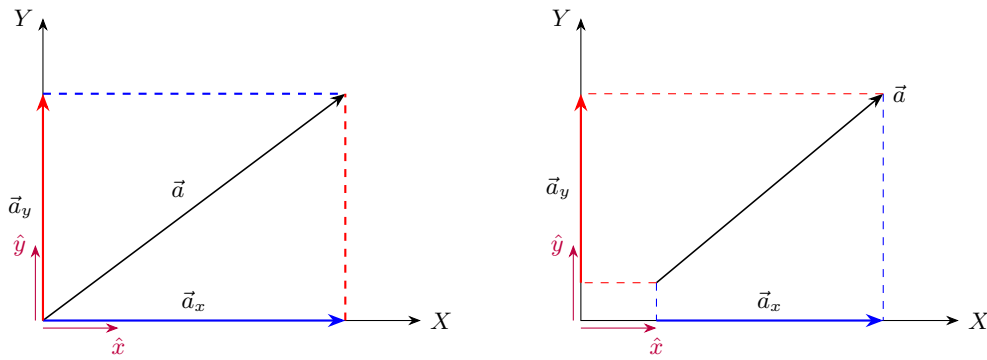


Vector projections

- The *projection* of a vector \vec{a} in the e direction is indicated by \vec{a}_e and defined as $\vec{a}_e = a_e \hat{e}$, that is,
 - its magnitude is a_e , the component of \vec{a} in the e direction; and
 - its direction is the same as that of the vector \hat{e} .



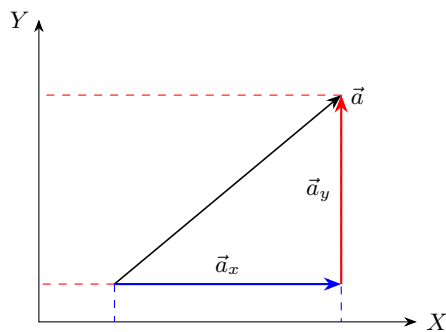
- Note the projection \vec{a}_x of the vector \vec{a} in the x direction and the projection \vec{a}_y of the vector \vec{a} in the y direction (in two-dimensional Cartesian reference system) in the following figures:



Clearly, we have

$$\vec{a}_x = a_x \hat{x} \quad \text{and} \quad \vec{a}_y = a_y \hat{y}.$$

We could also translate these projection vectors as follows:

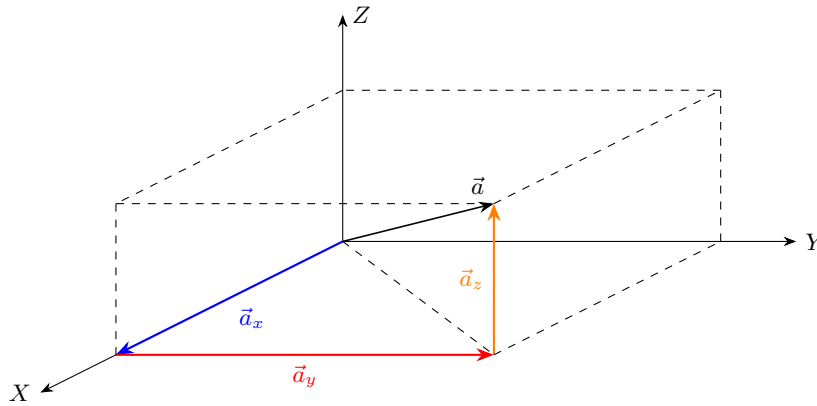


By the laws of vector addition it now follows that

$$\vec{a} = \vec{a}_x + \vec{a}_y = a_x \hat{x} + a_y \hat{y}.$$

This is called the *component form* of the vector \vec{a} .

- Consider now the following figure of a vector in the three-dimensional Cartesian reference system:

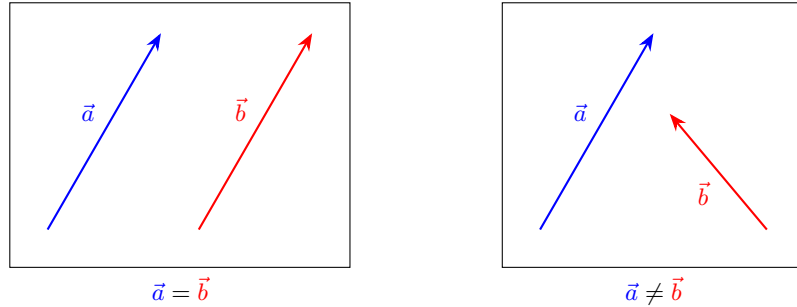


Again, it follows from vector addition that

$$\vec{a} = \vec{a}_x + \vec{a}_y + \vec{a}_z = a_x \hat{x} + a_y \hat{y} + a_z \hat{z}.$$

Vector algebra

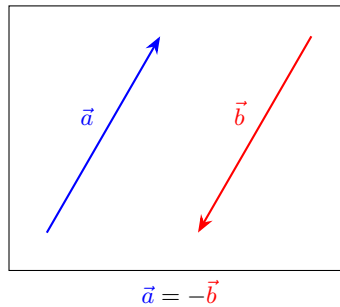
- Two vectors \vec{a} and \vec{b} are *equal*, indicated by $\vec{a} = \vec{b}$, if they have the same direction and magnitude.



If $\vec{a} = a_x\hat{x} + a_y\hat{y} + a_z\hat{z}$ and $\vec{b} = b_x\hat{x} + b_y\hat{y} + b_z\hat{z}$, then $\vec{a} = \vec{b}$ if and only if

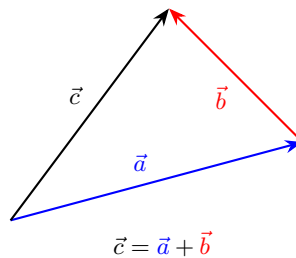
$$a_x = b_x, \quad a_y = b_y, \quad a_z = b_z.$$

- Two vectors \vec{a} and \vec{b} that have the same magnitude but opposite (and parallel) are called *opposites* of each other, indicated by $\vec{a} = -\vec{b}$.



In the figure above, \vec{a} is also called the *negative* of \vec{b} .

- The sum of two vectors \vec{a} and \vec{b} , called the *vector sum*, is a third vector \vec{c} . This sum is indicated by $\vec{c} = \vec{a} + \vec{b}$.



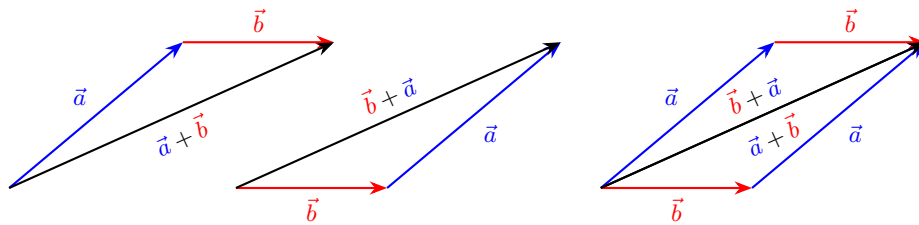
If $\vec{a} = a_x\hat{x} + a_y\hat{y} + a_z\hat{z}$, $\vec{b} = b_x\hat{x} + b_y\hat{y} + b_z\hat{z}$ and $\vec{c} = c_x\hat{x} + c_y\hat{y} + c_z\hat{z}$, then $\vec{c} = \vec{a} + \vec{b}$ if and only if

$$c_x = a_x + b_x, \quad c_y = a_y + b_y, \quad c_z = a_z + b_z.$$

The algebraic properties of the vector sum are:

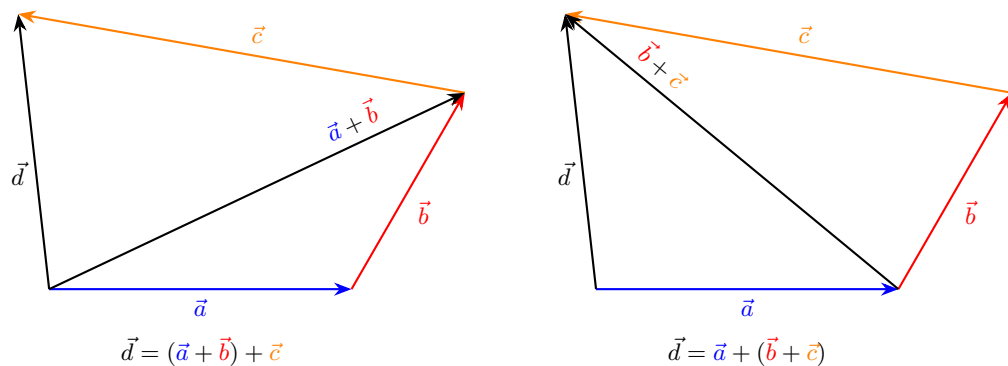
- the vector sum is *closed*, that is, the sum is a vector itself;
- the vector sum is *commutative*, that is,

$$\vec{a} + \vec{b} = \vec{b} + \vec{a};$$



- the vector sum is *associative*, that is,

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}).$$



- The *null vector*, indicated by $\vec{0}$, is obtained from the vector sum of a vector and its own negative, that is,

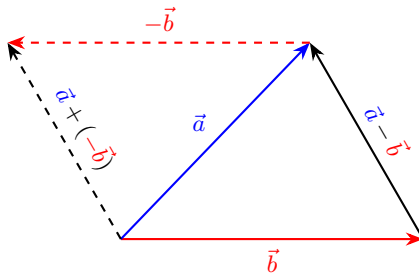
$$\vec{a} + (-\vec{a}) = \vec{0}.$$

The null vector is a vector with magnitude which is zero and the direction of which cannot be determined. Any vector remains unchanged when the null vector is added to it, that is,

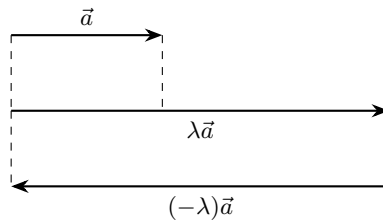
$$\vec{a} + \vec{0} = \vec{a}.$$

- The difference between two vectors \vec{a} and \vec{b} , called the *vector difference* and indicated by $\vec{a} - \vec{b}$, is defined as

$$\vec{a} - \vec{b} = \vec{a} + (-\vec{b}).$$



- The product of a scalar $\lambda > 0$ and a vector \vec{a} , indicated by $\lambda\vec{a}$, is a vector defined as follows:
 - $\lambda\vec{a}$ is parallel to \vec{a} , indicated as $\lambda\vec{a} \parallel \vec{a}$, that is, it has the same direction as \vec{a} ;
 - the magnitude of $\lambda\vec{a}$ is given by $|\lambda\vec{a}| = \lambda|\vec{a}|$.



If $\vec{a} = a_x\hat{x} + a_y\hat{y} + a_z\hat{z}$, then

$$\begin{aligned}\lambda\vec{a} &= \lambda(a_x\hat{x} + a_y\hat{y} + a_z\hat{z}) \\ &= (\lambda a_x)\hat{x} + (\lambda a_y)\hat{y} + (\lambda a_z)\hat{z}.\end{aligned}$$

Note that

$$(-\lambda)\vec{a} = -(\lambda\vec{a})$$

and

$$0\vec{a} = \vec{0}.$$

The algebraic properties of this product are:

$$\begin{aligned}\lambda(\mu\vec{a}) &= (\lambda\mu)\vec{a} && \text{(associativity)} \\ \lambda(\vec{a} + \vec{b}) &= \lambda\vec{a} + \lambda\vec{b} && \text{(distributivity)}\end{aligned}$$