Applied Mathematics APM01A1, 2017

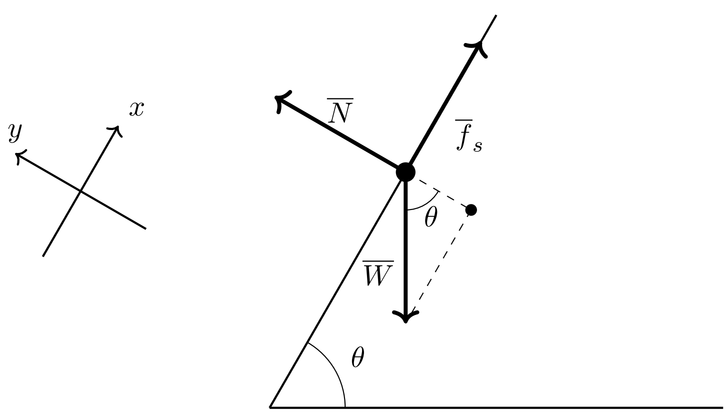
May 4, 2017

Tutorial 8

Question 1

A block with weight 50 N rests on a rough flat surface. The surface is required to be inclined by $60°$ from the horizontal. Calculate the coefficient of static friction μ_s the surface would need. If the surface is required to increase its incline, would you expect μ_s to increase or decrease?

Solution



The equilibrium condition is $\overline{N} + \overline{W} + \overline{f}_s = \overline{0}$. The forces along the x and y directions are

$$-W \sin(60°) + \mu_s N = 0 \quad (1)$$
$$N - W \cos(60°) = 0. \quad (2)$$

From equation (2) we obtain $N = W \cos(60°)$. Substituting back into (1), we obtain

$$-W \sin(60°) + \mu_s W \cos(60°) = 0.$$

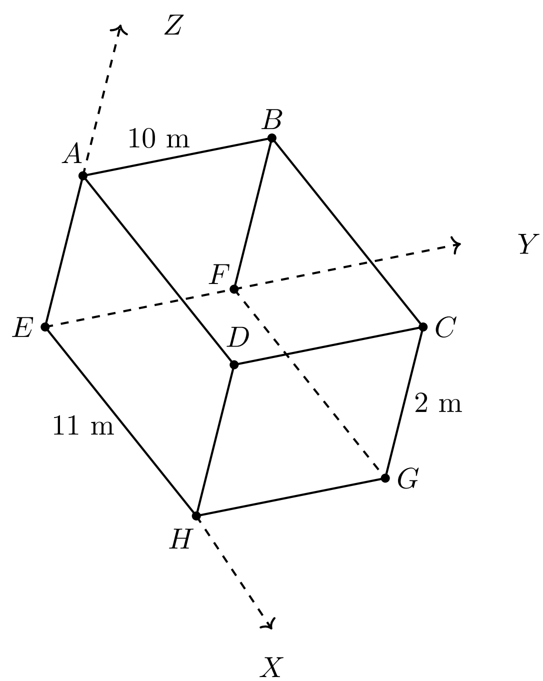
Solving for μ_s, we obtain

$$\mu_s = \tan(60^\circ) = \sqrt{3}.$$

As we can see from the above equation, if θ increases, μ_s would also increase. If $\theta = 90^\circ$, then the mass would fall to the ground, and we would need an infinite μ_s.

Question 2

In the figure below, a 100 N force is applied to the box at point C in the direction of point F. Calculate the moment generated by this force about point E.



Solution

To obtain the force in component form, begin by labelling the points C and F:

$$C = (11, 10, 2), \qquad F = (0, 10, 0)$$

The unit vector associated with line segment $\overline{CF}$ is

$$\frac{\overline{CF}}{|\overline{CF}|} = -\frac{1}{5\sqrt{5}}\left(11\hat{x} + 2\hat{z}\right).$$

This is the direction of the force $\overline{F}$. Thus, the 100 N force in component form is

$$\overline{F} = -\frac{100}{5\sqrt{5}}\left(11\hat{x} + 2\hat{z}\right)\ \text{N}.$$

Next, we need to find the position vector from E to the point of application of the force (point C). The position vector is

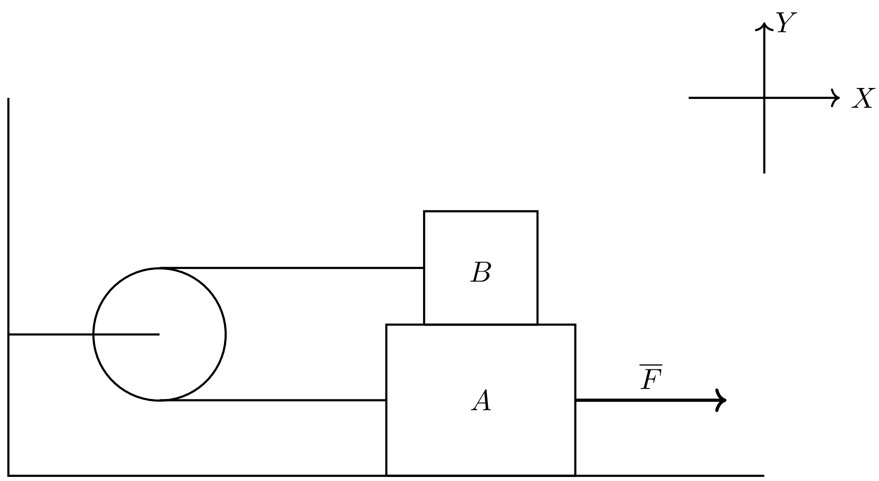
$$\overline{r} = 11\hat{x} + 10\hat{y} + 2\hat{z}.$$

To calculate the moment, we compute

$$\overline{M}_E = \overline{r} \times \overline{F} = -80\sqrt{5}\hat{x} + 440\sqrt{5}\hat{z}.$$

Question 3

In the figure below, let the weight of mass A be 2 N and the weight of mass B be 1 N. The coefficient of static friction μ_s is the same between all surfaces. Let the magnitude of force $\overline{F}$ be 5 N. If the system is just about to start moving, calculate μ_s.



Solution

First, draw a free body diagram for each block. For block A, there are seven forces acting. The equilibrium condition is

$$\overline{F} + \overline{f}_{sA} + \overline{f}_{sB} + \overline{T} + \overline{N}_A + \overline{W}_A + \overline{W}_B = \overline{0}$$

In the above, $\overline{f}_{sA}$ is the friction force between A and the ground, while $\overline{f}_{sB}$ is the friction force between A and B. Separating this equation into two equations (one for the x-component and one for the y-component), we find

$$\begin{aligned} F - T - \mu_s N_{A+B} - \mu_s N_B &= 0 \\ N_A - W_A - W_B &= 0. \end{aligned}$$

From the second equation, we obtain $N_{A+B} = (1+2)$ N $= 3$ N. Then, substituting the appropriate values into the first equation, we find

$$5 - T - 3\mu_s - N_B\mu_s = 0.$$

Performing the same analysis of block B, the equilibrium condition is

$$\overline{T} + \overline{f}_{sB} + \overline{N}_B + \overline{W}_B = \overline{0}.$$

Separating this equation into two equations (one for the x-component and one for the y-component), we find

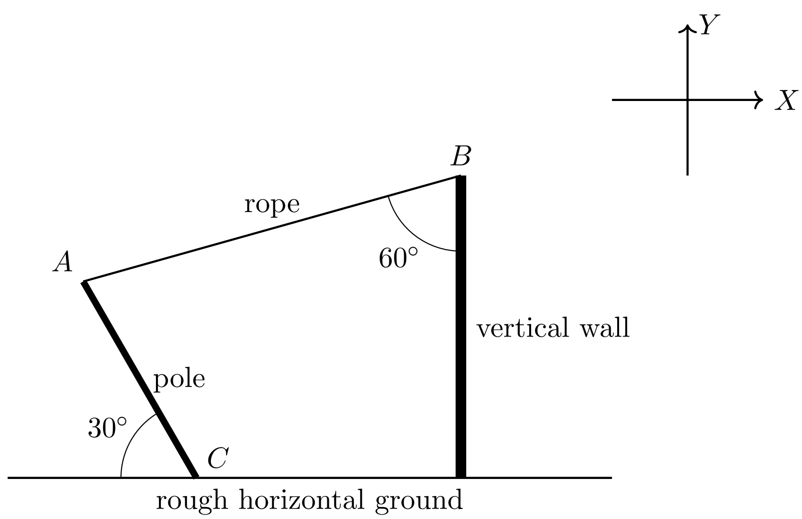
$$\begin{aligned} -T + \mu_s N_B &= 0 \\ N_B - W_B &= 0. \end{aligned}$$

From the second equation, $N_B = W_B = 1$ N. Then from the first equation $T = \mu_s$. We may now solve for μ_s

$$5 - 1\mu_s - 3\mu_s - 1\mu_s = 0, \qquad \mu_s = 1.$$

Question 4

The figure below shows an inclined uniform pole AC of weight 200 N held by a rope which is tied at A and B (all in the same vertical plane). The other end C of the pole rests on rough horizontal ground. **Hint**: One may model the weight of the pole as acting at its midpoint and directed straight downwards.



4.a) Calculate the tension in the rope.

4.b) Calculate the reaction force at point C in component form.

Solution

4.a) To calculate the tension, calculate the moment about point C. This is essentially a two-dimensional problem and any moments calculated will either point out of the page or into the page depending on the vector product. A clockwise rotation leads to a negative moment and an anti-clockwise rotation leads to a positive moment. Thus, the equilibrium condition for the moment is

$$M_C = -Tl\sin(60^\circ) + W\frac{l}{2}\sin(60^\circ) = 0,$$

leading to $T = W/2 = 100$ N.

4.b) The forces acting on the pole are the tension

$$\overline{T} = 100\cos(30^\circ)\,\hat{x} + 100\sin(30^\circ)\,\hat{y},$$

the weight of the pole

$$\overline{W} = -200\hat{y},$$

and finally, a reaction force $\overline{R}$ at C which is unknown. Note that the static friction force is included in this reaction force. Summing up the x-components and equating them to zero, we find

$$R_x = 100\cos(30^\circ) = 50\sqrt{3}\ \text{N}.$$

Lastly, summing up the y-components, we find

$$100\sin(30^\circ) - 200 + R_y = 0, \qquad R_y = 200 - 100\sin(30^\circ) = 150\ \text{N}.$$