Applied Mathematics APM01A1, 2017

March 27, 2017

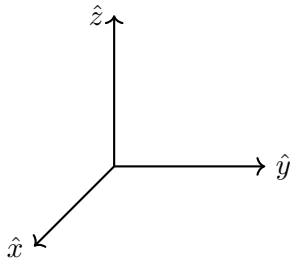
Tutorial 6

Question 1

The vertices of a triangle have $\overline{a}, \overline{b}$ and $\overline{c}$ as position vectors. Show that the angle θ at the vertex with position vector $\overline{c}$ is given by

$$\cos(\theta) = \frac{(\overline{a} - \overline{c}) \cdot (\overline{b} - \overline{c})}{|(\overline{a} - \overline{c}) \cdot (\overline{a} - \overline{c})(\overline{b} - \overline{c}) \cdot (\overline{b} - \overline{c})|^{1/2}}$$

Question 2



Use the figure above and the right-hand rule to evaluate the following

2.a) $\hat{x} \times \hat{y}$

2.b) $\hat{y} \times \hat{x}$

2.c) $\hat{y} \times (-\hat{x})$

2.d) $(-\hat{x}) \times \hat{y}$

2.e) $\hat{x} \times \hat{z}$

2.f) $\hat{z} \times \hat{y}$

Question 3

Find the area of the triangle which has vertices at the points $(1, 3, 2)$, $(2, -1, 1)$ and $(-1, 2, 3)$ using the vector product.

Question 4

Consider two vectors $\overline{a} = 6\hat{x}$ and $\overline{b} = \cos(\theta)\hat{x} + \sin(\theta)\hat{y}$, where θ is measured counterclockwise from the positive x direction.

4.a) Is $\overline{b}$ a unit vector. Justify your answer?

4.b) Let $0 < \cos(\theta)$ and $0 < \sin(\theta)$. In what direction does $\overline{a} \times \overline{b}$ point?

4.c) Let $\cos(\theta) > 0$ and $0 > \sin(\theta)$. In what direction does $\overline{a} \times \overline{b}$ point?

Question 5

Show that

$$\left(\overline{a} \times \overline{b}\right) \cdot \left(\overline{a} \times \overline{b}\right) = \left(\overline{a} \cdot \overline{a}\right)\left(\overline{b} \cdot \overline{b}\right) - \left(\overline{a} \cdot \overline{b}\right)^2$$

Question 6

Let $\overline{a} = 3\hat{x} + 2\hat{y} + \hat{z}, \overline{b} = \hat{x} - 2\hat{y} + \hat{z}$ and $\overline{c} = \hat{x} - \hat{y} + 2\hat{z}$.

6.a) Calculate $\left(\overline{a} \times \overline{b}\right) \times \overline{c}$

6.b) Is vector $\overline{a}$ perpendicular to $\left(\overline{a} \times \overline{b}\right) \times \overline{c}$? Justify your answer by performing an appropriate calculation.

6.c) Is vector $\overline{c}$ perpendicular to $\left(\overline{a} \times \overline{b}\right) \times \overline{c}$? Justify your answer by performing an appropriate calculation.