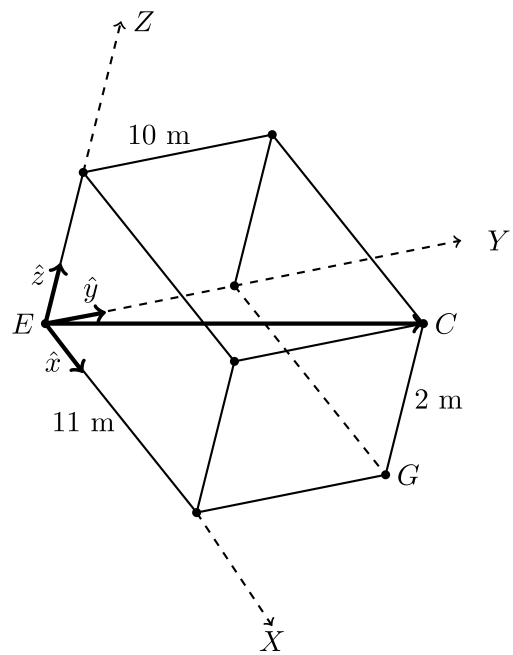
# Applied Mathematics APM01A1, 2017

March 13, 2017

## Tutorial 5

### Question 1

Study the following figure




1.a)

$$\overline{a} = 11\hat{x} + 10\hat{y} + 2\hat{z}$$

Next, we take the scalar product of $\overline{a}$ with $\{\hat{x}, \hat{y}, \hat{z}\}$.

$$\begin{aligned}\overline{a}\cdot\hat{x} &= 11(\hat{x}\cdot\hat{x}) = 15\cos(\alpha)\\ \overline{a}\cdot\hat{y} &= 10(\hat{y}\cdot\hat{y}) = 15\cos(\beta)\\ \overline{a}\cdot\hat{x} &= 2(\hat{z}\cdot\hat{z}) = 15\cos(\gamma)\end{aligned}$$

where each of the scalar products on the right-hand-side are equal to 1. Thus, solving for the angles

$$\begin{aligned}\alpha &= 42.83^o\\ \beta &= 48.19^o\\ \gamma &= 82.34^o.\end{aligned}$$

1.b) Finding the unit vector $\hat{e}$

$$\begin{aligned}\hat{e} &= \frac{\overline{EG}}{|\overline{EG}|}\\ &= \frac{1}{\sqrt{(11)^2+(10)^2}}(11\hat{x}+10\hat{y})\\ &= \frac{11}{\sqrt{221}}\hat{x}+\frac{10}{\sqrt{221}}\hat{y}\end{aligned}$$

Then calculate the scalar product

$$\begin{aligned}\overline{a}\cdot\hat{e} &= (11\hat{x}+10\hat{y}+2\hat{z})\cdot\left(\frac{11}{\sqrt{221}}\hat{x}+\frac{10}{\sqrt{221}}\hat{y}\right)\\ &= \sqrt{221}.\end{aligned}$$

Thus, the component of $\overline{a}$ along the $\hat{e}$ direction is $a_e = \sqrt{221}$.

## Question 2

Let $\overline{a}$ and $\overline{b}$ be vectors with magnitudes 24 and 20 respectively. Suppose $\overline{c} = \overline{a} + \overline{b}$ and $\overline{c}$ makes an angle $\theta$ with $\overline{b}$, where $\cos(\theta) = 1/10$.

2.a) Solving for $\overline{a}$,

$$\overline{a} = \overline{c} - \overline{b}.$$

Now square both sides using the scalar product

$$\begin{aligned}\overline{a}\cdot\overline{a} &= (\overline{c}-\overline{b})\cdot(\overline{c}-\overline{b})\\ |\overline{a}|^2 &= |\overline{c}|^2 + |\overline{b}|^2 - 2|\overline{b}||\overline{c}|\cos(\theta)\\ &= |\overline{c}|^2 + |\overline{b}|^2 - \frac{2|\overline{b}||\overline{c}|}{10}\end{aligned}$$

2.b) Solving for $|\overline{c}|$,

$$\begin{aligned} 0 &= |\overline{c}|^2 - \frac{2|\overline{b}||\overline{c}|}{10} - |\overline{a}|^2 + |\overline{b}|^2 \\ 0 &= |\overline{c}|^2 - 4|\overline{c}| - 176 \end{aligned}$$

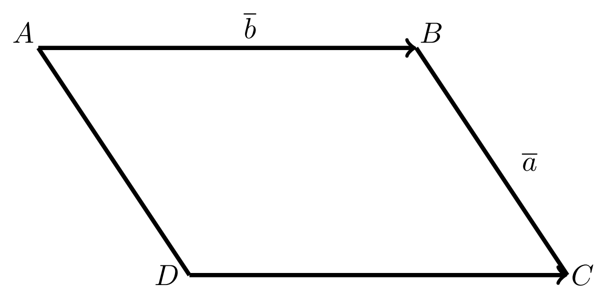
The solution to this equation is

$$|\overline{c}| = 2(1 - 3\sqrt{5}), \qquad \text{or} \qquad |\overline{c}| = 2(1 + 3\sqrt{5})$$

We take the positive root for the magnitude, $|\overline{c}| = 2(1 + 3\sqrt{5})$.

## Question 3

Suppose that $ABCD$ is a parallelogram such that $\overline{b} = \overline{AB} = 6\hat{x} + 2\hat{y} - 3\hat{z}$ and $\overline{c} = \overline{AC} = 8\hat{x} + 6\hat{y} - 2\hat{z}$. Let $\overline{a} = \overline{BC}$.




2.a) From the previous tutorial,

$$\overline{a} = 2\hat{x} + 4\hat{y} + \hat{z}$$

. To find $\angle ABC$, calculate

$$\begin{aligned} \overline{BA} \cdot \overline{BC} &= -\overline{b} \cdot \overline{a} \\ -17 &= 7\sqrt{21}\cos(\theta) \\ \theta &= 122^o \end{aligned}$$

Next, for $\angle ADC$, calculate

$$\begin{aligned} \overline{DA} \cdot \overline{DC} &= -\overline{a} \cdot \overline{b} \\ -17 &= 7\sqrt{21}\cos(\theta) \\ \theta &= 122^o \end{aligned}$$

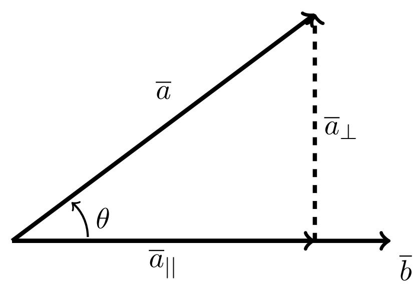
2.b) First, for $\angle DAB$, calculate

$$\begin{aligned}\overline{AB}\cdot\overline{AD} &= \overline{b}\cdot\overline{a} \\ 17 &= 7\sqrt{21}\cos(\theta) \\ \theta &= 58^o\end{aligned}$$

Finally, for $= \angle DCB$, calculate

$$\begin{aligned}\overline{CD}\cdot\overline{CB} &= +\overline{b}\cdot\overline{a} \\ 17 &= 7\sqrt{21}\cos(\theta) \\ \theta &= 58^o\end{aligned}$$

## Question 4




Note that

$$\begin{aligned}|\overline{a}| &= \sqrt{6} \\ |\overline{b}| &= 9 \\ \hat{b} &= \frac{4}{9}\hat{x} - \frac{4}{9}\hat{y} + \frac{7}{9}\hat{z}\end{aligned}$$

The projection of $\overline{a}$ in the direction of $\overline{b}$, or equivalently $\hat{b}$, is given by

$$\overline{a}_{||} = \sqrt{6}\cos(\theta)\hat{b}.$$

To find $\theta$, compute the scalar product $\overline{a}\cdot\overline{b}$ to find

$$\theta = \arccos\left(\frac{19}{9\sqrt{6}}\right) \approx 30.47^o$$

Thus,

$$\overline{a}_{||} = \frac{76}{81}\hat{x} - \frac{76}{81}\hat{y} + \frac{133}{81}\hat{z}$$

Next, we need to find the vector perpendicular to $\overline{b}$. We can write $\overline{a}$ as

$$\overline{a} = \overline{a}_{||} + \overline{a}_\perp$$

Thus,

$$\overline{a}_{\perp} = \frac{5}{81}\hat{x} - \frac{86}{81}\hat{y} - \frac{52}{81}\hat{z}.$$

Once can verify that

$$\begin{aligned} |\overline{a}_{\perp}| &= \sqrt{6}\sin(\theta) \\ \overline{a}\cdot\overline{b} &= 0. \end{aligned}$$