Applied Mathematics APM01A1, 2017

March 10, 2017

Tutorial 4 Solutions

Question 1

1.a) The components of vector $\overline{a}$:

$$\begin{aligned} a_x &= 11 \\ a_y &= 10 \\ a_z &= 2. \end{aligned}$$

1.b) The projection of $\overline{a}$ in the x, y and z directions. Note that the projections themselves are vectors, so of course, we need to write them as such:

$$\begin{aligned} \overline{a}_x &= 11\hat{x} \\ \overline{a}_y &= 10\hat{y} \\ \overline{a}_z &= 2\hat{z}. \end{aligned}$$

1.c)

$$\overline{a} = 11\hat{x} + 10\hat{y} + 2\hat{z}.$$

1.d) The magnitude of $\overline{a}$:

$$|\overline{a}| = \sqrt{(10)^2 + (11)^2 + (2)^2} = 15.$$

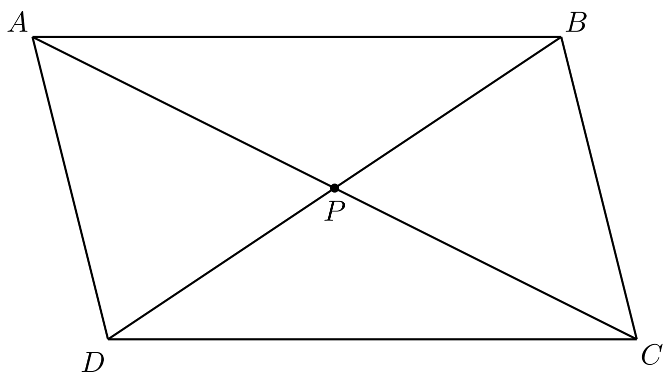
1.e) To find the projection of $\overline{a}$ in the $\overline{EG}$ direction, we need to find the angle θ that $\overline{a}$ makes with this projection. This angle is simply 90^o minus the angle that $\overline{a}$ makes with the z-axis. For this, we need to the direction cosine in the z direction:

$$n = \cos(\gamma) = \frac{2}{15} \quad \Rightarrow \quad \gamma = 82.33^o.$$

Thus $\theta = 7.66^o$. Now one may find the projection

$$\overline{a_e} = |\overline{a}|\cos(\theta)\hat{e} = 15\cos(7.66)\hat{e} = 14.87\hat{e}.$$

Question 2



2.a) Since $ABCD$ is a parallelogram, we have vector

$$\overline{DC} = \overline{AB} = \overline{b} = 6\hat{x} + 2\hat{y} - 3\hat{z}.$$

2.b) Next, we observe that $\overline{BC} = \overline{c} - \overline{b}$. In component form

$$\overline{BC} = \overline{c} - \overline{b} = 2\hat{x} + 4\hat{y} + \hat{z}.$$

2.c) Next, we observe that $\overline{DB} = \overline{b} + \overline{CB}$. In component form

$$\overline{DB} = \overline{b} + \overline{CB} = 4\hat{x} - 2\hat{y} - 4\hat{z}.$$

2.d) The diagonals of a parallelogram bisect each other. Thus,

$$\begin{aligned}
\overline{BC} &= \overline{BP} + \overline{PC} \\
&= \frac{1}{2}\overline{BD} + \frac{1}{2}\overline{c} \\
&= \frac{1}{2}((-4\hat{x} + 2\hat{y} + 4\hat{z}) + (8\hat{x} + 6\hat{y} - 2\hat{z})) \\
&= 2\hat{x} + 4\hat{y} + \hat{z}.
\end{aligned}$$

Question 3

3.a) The magnitudes of the position vectors $\overline{r}_1$ and $\overline{r}_2$ are

$$\begin{aligned}
|\overline{r}_1| &= \sqrt{(2)^2 + (4)^2 + (2)^2} = \sqrt{24}\text{ m}, \\
|\overline{r}_2| &= \sqrt{(-3)^2 + (1)^2 + (-4)^2} = \sqrt{26}\text{ m}.
\end{aligned}$$

3.b) The unit vector of $\overline{r}_1$ in component form is

$$\begin{aligned}\hat{r}_1 &= \frac{\overline{r}_1}{|\overline{r}_1|} = \frac{2}{\sqrt{24}}\hat{x} + \frac{4}{\sqrt{24}}\hat{y} + \frac{2}{\sqrt{24}}\hat{z} \\ &= \cos(65.9^o)\hat{x} + \cos(35.26^o)\hat{y} + \cos(65.9^o)\hat{z}\end{aligned}$$

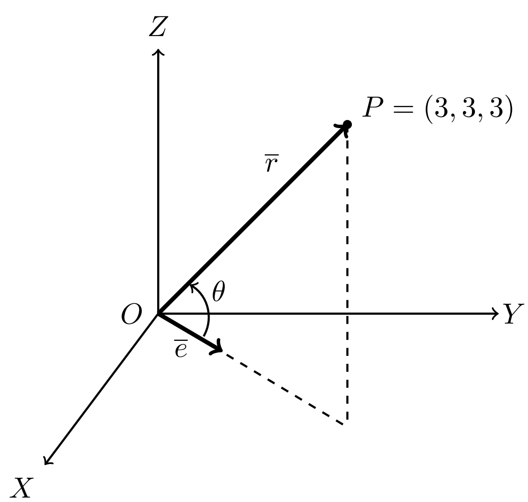
3.c) Here, we need to calculate the magnitude of vector $\overline{P_1P_2}$, $|\overline{P_1P_2}| = |\overline{r}_2 - \overline{r}_1|$

$$\begin{aligned}\overline{P_1P_2} &= \overline{r}_2 - \overline{r}_1 \\ &= -5\hat{x} - 3\hat{y} - 6\hat{z}.\end{aligned}$$

Thus, the magnitude is

$$|\overline{P_1P_2}| = \sqrt{(-5)^2 + (-3)^2 + (-6)^2} = \sqrt{70}\text{ m}.$$

Question 4



4.a) No, $\overline{e}$ is not a unit vector since its magnitude is not equal to 1:

$$|\overline{e}| = \sqrt{1^2 + 1^2} = \sqrt{2}.$$

4.b) The position vector $\overline{r}$ is

$$\overline{r} = 3\hat{x} + 3\hat{y} + 3\hat{z}.$$

The magnitude of position vector $\overline{r}$ is

$$|\overline{r}| = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{27}.$$

4.c) We need to calculate the angle between vectors $\overline{e}$ and $\overline{r}$. This is simply the angle of elevation and may be found from the z-component $\overline{r}$, denoted by r_z, and the magnitude of $\overline{r}$, denoted by r:

$$\sin(\theta) = \frac{r_z}{r} = \frac{3}{\sqrt{27}} \quad \Rightarrow \quad \theta = 35.26^o.$$

To find the component of $\overline{r}$ in the $\overline{e}$ direction, we just need to calculate

$$r_e = |\overline{r}| \cos(\theta) = 3\sqrt{2}.$$

4.d) The projection of $\overline{r}$ in the direction of $\overline{e}$ is simply its component times by the unit vector $\hat{e}$. The unit vector $\hat{e}$ is equal to

$$\hat{e} = \frac{1}{\sqrt{2}}(\hat{x} + \hat{y}).$$

Next, the projection is equal to

$$\overline{r}_e = r_e \hat{e} = 3\hat{x} + 3\hat{y}.$$