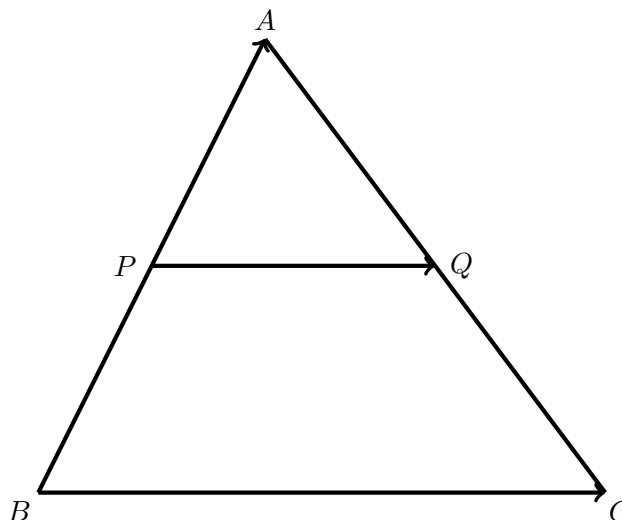


Applied Mathematics APM01A1, 2017

February 27, 2017

Tutorial 3 Solutions

Question 1



First, for \overline{BC} and \overline{PQ} , we can write:

$$\begin{aligned}\overline{BC} &= \overline{BA} + \overline{AC} \\ \overline{PQ} &= \overline{PA} + \overline{AQ}.\end{aligned}$$

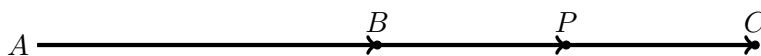
Since, the ratio of $|\overline{AP}|$ to $|\overline{PB}|$ and of $|\overline{AQ}|$ to $|\overline{QC}|$ is 2 : 3, we have

$$\overline{PA} = \frac{2}{5}\overline{BA}, \quad \text{and} \quad \overline{AQ} = \frac{2}{5}\overline{AC},$$

it follows that

$$\overline{PQ} = \frac{2}{5}(\overline{BA} + \overline{AC}) = \frac{2}{5}\overline{BC}.$$

Question 2



First note that

$$\begin{aligned}\overline{BP} &= \overline{AP} - \overline{AB} \\ &= \overline{AP} - \frac{4}{7}\overline{AC}.\end{aligned}$$

But $\overline{AC} = \overline{PC} + \overline{AP}$, so that

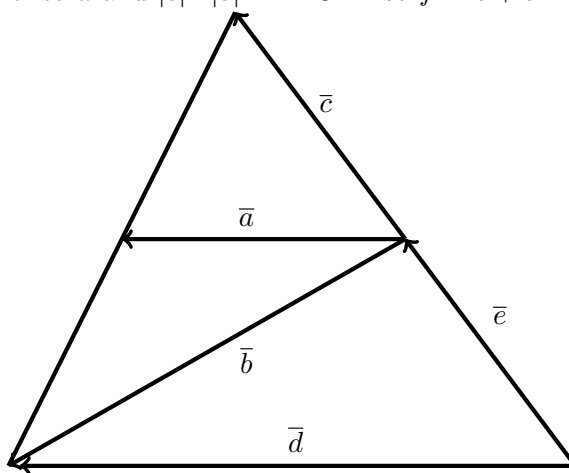
$$\begin{aligned}\overline{BP} &= \overline{AP} - \frac{4}{7}(\overline{PC} + \overline{AP}) \\ &= \frac{3}{7}\overline{AP} - \frac{4}{7}\overline{PC}.\end{aligned}$$

This is just equivalent to

$$\overline{PB} = \frac{3}{7}\overline{PA} + \frac{4}{7}\overline{PC}.$$

Question 3

In the following figure, \bar{a} is parallel to \bar{d} and $|\bar{c}| : |\bar{e}| = 2 : 3$. Also $\bar{f} = \bar{c} + \bar{e}$.



3.a)

$$\bar{a} = \alpha \bar{b} + (1 - \alpha) \bar{c} = \alpha \bar{b} + \frac{2}{5}(1 - \alpha) \bar{f}$$

OR one could have written

$$\bar{a} = \alpha\bar{c} + (1 - \alpha)\bar{b} = \frac{2\alpha}{5}\bar{f} + (1 - \alpha)\bar{b}$$

3.b)

$$\bar{d} = \bar{e} + \bar{b} = \frac{3}{5}\bar{f} + \bar{b}.$$

3.c) First note that $\bar{a} = k\bar{d}$. Then use parts a and b:

$$\begin{aligned}\bar{a} &= k\bar{d} \\ \alpha\bar{b} + \frac{2}{5}(1 - \alpha)\bar{f} &= k\left(\frac{3}{5}\bar{f} + \bar{b}\right)\end{aligned}$$

This means that

$$\alpha = k, \quad \text{and} \quad \frac{2}{5}(1 - \alpha) = \frac{3k}{5}$$

Thus, $k = 2/5$.

If the alternative expression for \bar{a} in part 3.a was used, then:

$$\frac{2\alpha}{5} = \frac{3k}{5}, \quad 1 - \alpha = k$$

and thus still find $k = 2/5$.

3.d) From the above, we see that $\alpha = 2/5$, OR $\alpha = 3/5$.