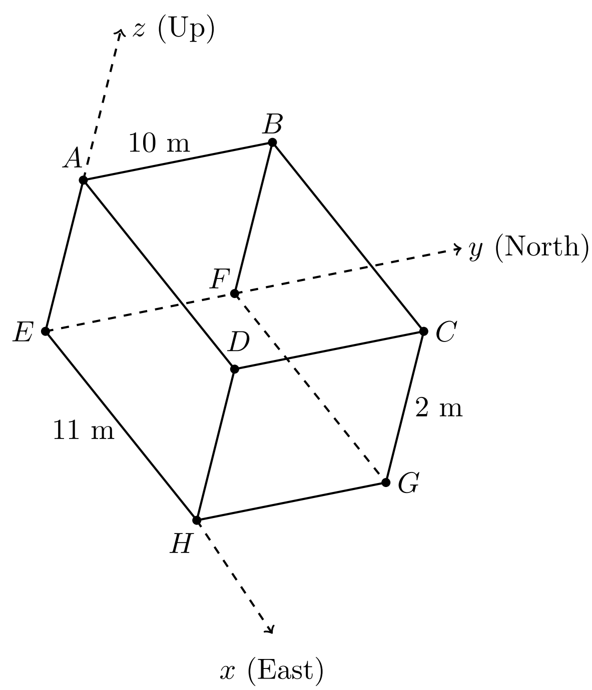
Applied Mathematics APM01A1, 2017

February 21, 2017

Tutorial 2 Solutions

Question 1

Consider the following rectangular box with the origin fixed at point E.



1.a)

$$\begin{aligned} A &= (0,0,2) \qquad & G = (11,10,0) \\ B &= (0,10,2) \qquad & H = (11,0,0) \\ C &= (11,10,2) \qquad & E = (0,0,0) \end{aligned}$$

1.b)

$$\begin{aligned}
x \text{ comp of } \overline{EC} &= 11 - 0 = 11\,\text{m} \\
y \text{ comp of } \overline{EC} &= 10 - 0 = 10\,\text{m} \\
z \text{ comp of } \overline{EC} &= 2 - 0 = 2\,\text{m}
\end{aligned}$$

$$\begin{aligned}
x \text{ comp of } \overline{HB} &= 11 - 0 = -11\,\text{m} \\
y \text{ comp of } \overline{HB} &= 10 - 0 = 10\,\text{m} \\
z \text{ comp of } \overline{HB} &= 2 - 0 = 2\,\text{m}
\end{aligned}$$

$$\begin{aligned}
x \text{ comp of } \overline{AG} &= 11 - 0 = 11\,\text{m} \\
y \text{ comp of } \overline{AG} &= 10 - 0 = 10\,\text{m} \\
z \text{ comp of } \overline{AG} &= 2 - 0 = -2\,\text{m}
\end{aligned}$$

1.c) We first need to calculate the magnitude of $\overline{AG}$.

$$|\overline{AG}| = \sqrt{11^2 + 10^2 + (-2)^2} = 15\,\text{m}$$

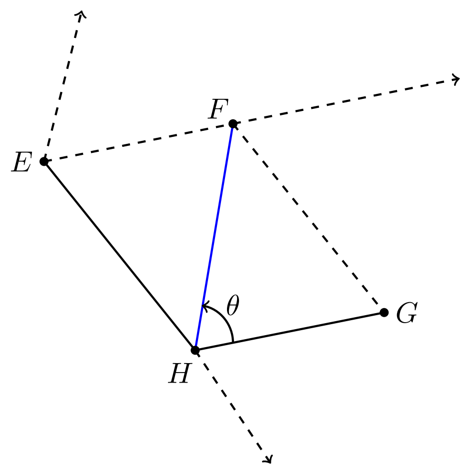
The direction cosines for the x, y and z directions are

$$\begin{aligned}
l &= cos(\alpha) = \frac{11}{15} \\
m &= cos(\beta) = \frac{10}{15} \\
n &= cos(\gamma) = \frac{-2}{15}
\end{aligned}$$

1.d) Here, we need to find the direction cosine for $\overline{AG}$ along the y-direction. The angle between $\overline{AG}$ and $\overline{AB}$ is simply β.

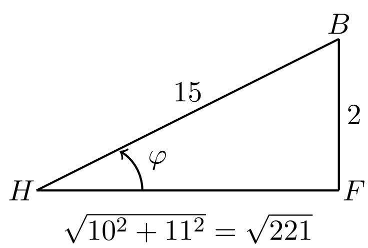
$$\beta = cos^{-1}\left(\frac{10}{15}\right) = 48.19^o.$$

1.e) For the cardinal angle, consider only the following portion of the above figure:



$$\tan(\theta) = \frac{11}{10} \quad \Rightarrow \quad \theta = 42.27^o \text{ North of West.}$$

and for the angle of elevation:



To calculate the φ, we simply do

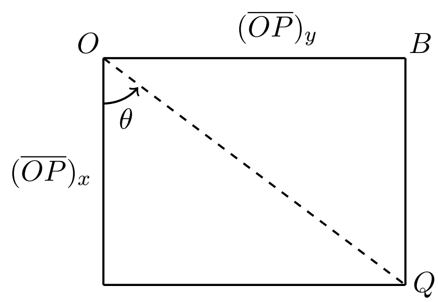
$$\varphi = \cos^{-1}(\sqrt{221}/15) = 7.66^o$$

Question 2

2.a) From the information given, i.e., $\angle POX = 60^o$ and $\angle POY = 70^o$, we are able to find the direction cosines along the x and y directions:

$$\begin{aligned} l &= \cos(60) = \frac{1}{2} = \frac{(\overline{OP})_x}{|\overline{OP}|} \\ m &= \cos(70) = 0.342 = \frac{(\overline{OP})_y}{|\overline{OP}|}, \end{aligned}$$

where $(\overline{OP})_x$ and $(\overline{OP})_y$ are the x and y components of $\overline{OP}$ respectively. From the figure below



we can calculate

$$\frac{m}{l} = \tan(\theta) = \frac{(\overline{OP})_y}{(\overline{OP})_x} \quad \Rightarrow \quad \theta = 34.37^o \text{ North of East}$$

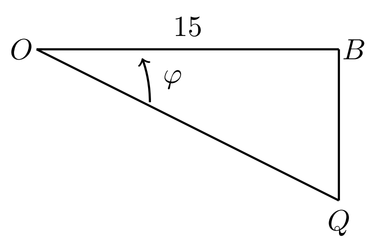
2.b) Using the result that the square of the direction cosines equals 1, $l^2 + m^2 + n^2 = 1$, we can solve for n, obtain the angle $\overline{OP}$ makes with the positive z direction, and hence obain the angle of elevation.

$$\begin{aligned} n &= 0.7956, \quad \Rightarrow \quad \gamma = \cos^{-1}(n) = 37.28^o \\ \text{angle of elevation} &= 90 - 37.28 = 52.71^o. \end{aligned}$$

2.c) Using Pythagoras, we can solve for $|\overline{OB}|$:

$$|\overline{OB}| = \sqrt{|\overline{OP}|^2 - |\overline{PB}|^2} = \sqrt{25^2 - 20^2} = 15\,\text{m}$$

Next, from triangle

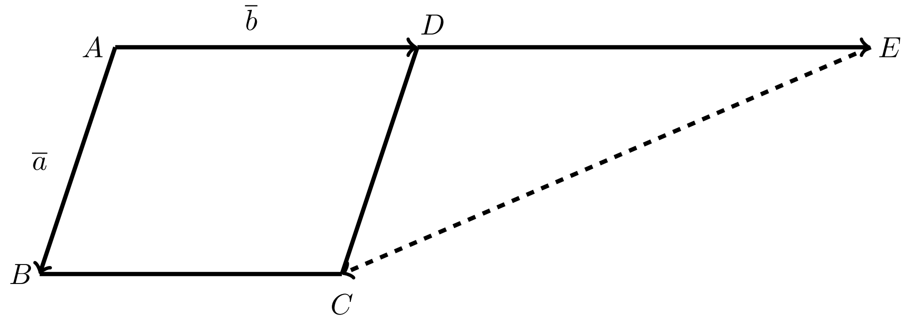


$$\cos(\varphi) = \cos(41.41) = \frac{15}{|\overline{OQ}|} \quad \Rightarrow \quad |\overline{OQ}| = 20\,\text{m}.$$

Finally, one can then find the x coordinate:

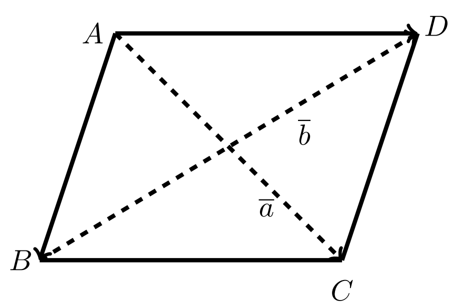
$$x = |\overline{OQ}|\sin(41.41) = 20\sin(41.41) = 13.23\,\text{m}.$$

Question 3



$$\begin{aligned}\overline{AE} &= 4\overline{b}\\ \overline{AC} &= \overline{a}+\overline{b}\\ \overline{EC} &= \overline{AC}-4\overline{b}\\ &= \overline{a}-3\overline{b}.\end{aligned}$$

Question 4



$$\begin{aligned}\overline{AB} &= \frac{\overline{a}}{2}-\frac{\overline{b}}{2}\\ \overline{BC} &= \frac{\overline{a}}{2}+\frac{\overline{b}}{2}\end{aligned}$$