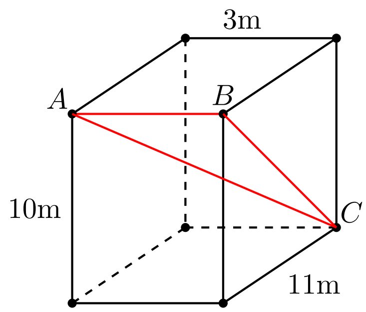
Applied Mathematics APM01A1, 2017

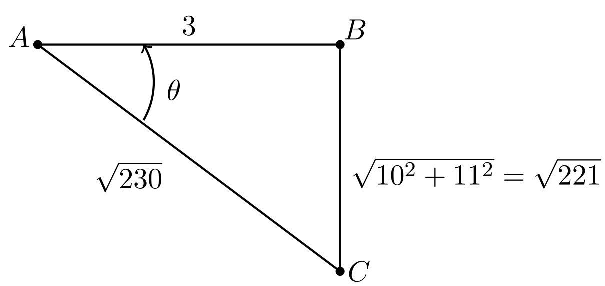
February 24, 2017

Tutorial 1 Solutions

Question 1



In the above figure, we recognise that $\Delta\ ABC$ is a right-angled triangle, i.e., $\angle ABC = 90^o$:

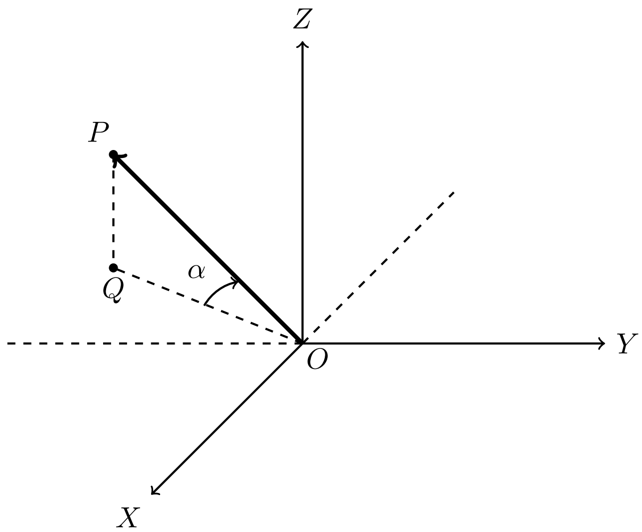


One may use the sin function to calculate one of the angles:

$$\sin(\theta) = \frac{\sqrt{221}}{\sqrt{230}} \quad \Rightarrow \quad \theta = 78.59^o.$$

The remaining angle is then simply $90 - \theta = 11.41^o$.

Question 2



2.a) We find the magnitude of the vector $\overline{OP}$:

$$|\overline{OP}| = \sqrt{(-5)^2 + (-14)^2 + (2)^2} = 15\,\text{m}.$$

2.b) The distance is simply the y coordinate $14\,\text{m}$.

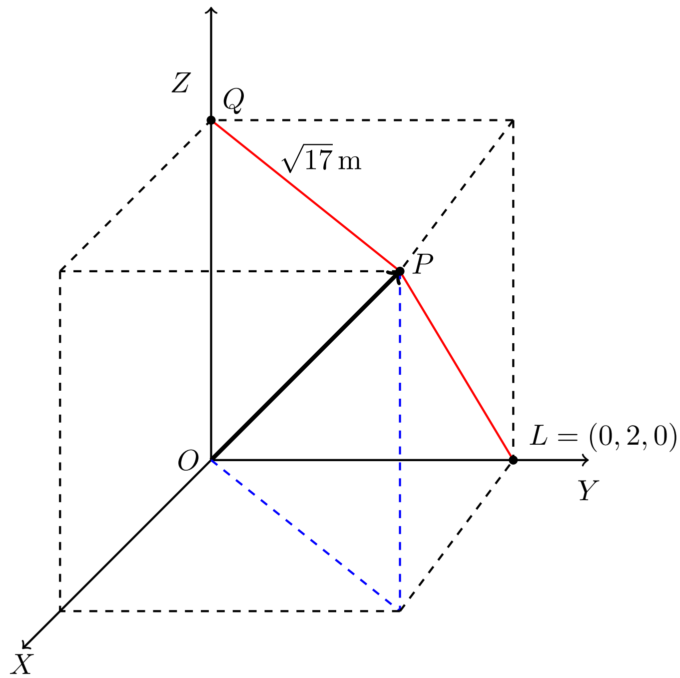
2.c) Here, we calculate the direction cosine along the x direction:

$$\cos(\theta) = \frac{-5}{15} \quad \Rightarrow \quad \theta = 109.47^o.$$

2.d) The angle of elevation, α, is found by studying the triangle ΔOQP. We use the cos function which requires us to find the distance $|\overline{OQ}|$:

$$\cos(\alpha) = \frac{\sqrt{(-5)^2 + (-14)^2}}{15} \quad \Rightarrow \quad \alpha = 7.66^o.$$

Question 3



3.a) This angle is simply the direction cosine in the y direction, which is written as $m = \cos(\beta)$ in the notes. To find this angle we first need to find the magnitude of $\overline{OP}$. Now, the distance from P to L is $\sqrt{60}$ m. Thus, we can use Pythagoras to find $|\overline{OP}|$.

$$|\overline{OP}| = \sqrt{60 + 4} = \sqrt{64} = 8\,\text{m}.$$

and hence

$$m = \cos(\beta) = \frac{2}{\sqrt{64}} \quad \Rightarrow \quad \beta = 75.52^o.$$

3.b) The distance from P to the z-axis is $\sqrt{17}$ m. Looking at triangle ΔOQP, we can use Pythagoras again to find $|\overline{OQ}|$, which is exactly the z coordinate we want. Thus,

$$|\overline{OQ}| = \sqrt{8^2 - 17} = \sqrt{47}\,\text{m}.$$

Question 4

4.a) The vector $\overline{AB}$ is formed from point A to point B. To find the components, we write:

$$\begin{aligned} x \text{ component of } \overline{AB} \text{ is } &= -1 - 3 = -4 \\ y \text{ component of } \overline{AB} \text{ is } &= 1 - (-2) = 3 \\ z \text{ component of } \overline{AB} \text{ is } &= 1 - 1 = 0 \end{aligned}$$

4.b) The magnitude is found from the formula

$$|\overline{AB}| = \sqrt{(-4)^2 + (3)^2 + 0^2} = 5\,\mathrm{m},$$

Question 5

This question simply asks for the magnitude of the resultant displacement vector.

$$\text{Magnitude} = \sqrt{100^2 + 700^2 + 300^2} = 100\sqrt{59}\,\mathrm{m}.$$